The Cumulative Capacitated Vehicle Routing Problem with Time-dependent on Humanitarian Logistics for Disaster Management

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Abstract

This study addresses the challenges of optimizing humanitarian logistics during disaster management by developing a Cumulative Capacitated Vehicle Routing Problem with Time-Dependent factors (CCVRP-TD) model. The primary objective is to enhance delivery efficiency by incorporating time-dependent variables such as fluctuating traffic and service durations into route planning. The research contributes a novel Mixed Integer Nonlinear Programming (MINLP) framework that dynamically adapts to real-world conditions like road closures and shifting priorities. Using the MINLP approach, the model was validated through numerical experiments involving four delivery vehicles serving six customers across five routes. Results demonstrated a significant improvement in routing efficiency, with a total cumulative travel distance of 110 km and adherence to specified delivery windows, such as 9:30 AM and 10:30 AM for Customer 1. Additionally, vehicle capacity constraints were effectively managed, with individual route lengths ranging from 20 to 35 km. These findings showcase the model's ability to balance cost minimization, service reliability, and logistical adaptability. The novelty lies in the integration of time-dependent costs and service benefits into a multi-depot framework, enabling flexible yet precise route optimization under constrained conditions. This research provides a robust tool for enhancing disaster logistics and offers practical implications for improving the responsiveness and effectiveness of humanitarian aid delivery.

Keywords: Humanitarian Logistics, Disaster Management, Vehicle Routing Problem, Time-Dependent Factors, Mixed Integer Nonlinear Programming

1. Introduction

Indonesia, along with other countries such as India and Haiti, is characterized by several regions highly susceptible to both natural and human-made disaster. This disaster occurs due to various factors, including geographical, geological, and climatic conditions, as well as social, cultural, and political diversity [1], [2]. To overcome these challenges, effective logistical planning is essential in the distribution of aid during disaster relief operations. Specifically, logistics plays an important role in assisting victims to determine the efficacy and success of the entire management operations. A significant aspect of disaster logistics is the strategic determination of emergency facility locations in post-disaster areas [1], [3], [4], [5], [6], [7], [8], [9]. These facilities serve as distribution centers for logistical aid, and the locations significantly impact the effectiveness of delivery routes. The proximity of the distribution centers to demand points (victims) is crucial to enhance fast delivery and increase the victims' chances of survival. In this context, advanced technologies and data analytics such as Vehicle Routing Problem (VRP) are increasingly being used to optimize logistics operations, enhance the precision, and responsiveness of disaster management efforts [2], [7], [9], [10], [11], [12].

The application of the VRP in disaster management is a crucial component of humanitarian logistics. Based on previous research, the earliest documented use of VRP to support relief efforts dates back to 1988 [13]. Since 2010, four comprehensive systematic literature reviews on Humanitarian Supply Chain (HSC) VRP have been conducted, covering a wide range of topics, including evacuation and rescue operations [14], modeling and optimizing relief operations [15], as well as addressing aid route challenges [14], [16]. The reviews predominantly focus on rapid-

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onset disaster such as hurricanes, earthquakes, and tsunamis, while slow-onset disaster including pandemics and droughts have not received adequate attention [14]. The results show the significant advancements and applications of VRP in improving disaster response efficiency. By optimizing routes for humanitarian aid, VRP ensures essential supplies reach affected areas swiftly, enhancing humanitarian aid. Additionally, its integration into evacuation and rescue operations has proven instrumental in saving lives by facilitating the organized movement of victims from disaster zones to safer locations. Despite the significant benefit, the gap in addressing slow-onset disaster presents an essential area for future research. Developing VRP models that adapt to the dynamic and extended timelines of events such as pandemics and droughts could significantly enhance the capability of humanitarian logistics, ensuring a more comprehensive and effective method of disaster management.

VRP can be formally represented through a directed graph $G=(V,A)$, where $V=\{v_1,v_2,\ldots,v_n\}$ denotes the set of nodes, and $A\subseteq\{(v_i,v_j):i\neq j;v_i,v_j\in V\}$ represents the set of arcs. The objective is to determine an optimal set of routes, each forming a cycle that starts and ends at the depot, to serve a collection of customers located at these nodes. Moreover, the primary objective is to minimize the total travel cost, which is typically proportional to travel time or distance, as well as the operational cost related to the number of vehicles [17], [18]. In addressing the VRP, various constraints must be considered, such as vehicle capacity, customer demand, and route length limitations. These constraints add complexity to the problem, requiring sophisticated optimization methods to solve effectively. The solution includes creating routes that ensure all customer demands are met without exceeding vehicle capacities while adhering to other logistical restrictions imposed [19].

The cumulative vehicle routing problem (CVRP) extends the classical capacitated VRP [20] by incorporating the accumulation of costs into the planning process. This extension aims to identify a set of delivery routes that optimally balances the total travel and operational costs, thereby achieving the most efficient distribution strategy under given constraints. The CVRP considers the immediate costs of individual routes and compounded costs over the entire planning horizon, providing a more holistic method of optimization. Moreover, advancements in computational algorithms and heuristics, such as genetic algorithms, simulated annealing, and ant colony optimization, have significantly enhanced the ability to determine near-optimal solutions for large and complex VRP instances. These methods combined with modern computational power, enable more efficient and practical applications of VRP solutions in real-world logistics and supply chain management scenarios.

The first cumulative VRP (Cum-VRP) was introduced in 2008 by [21] to integrate the flow of goods along the route in two key applications, namely minimizing energy consumption and routing school buses. In this method, the objective function is quantified as the product of the vehicle load and the arc cost traversed to reach the node where the requested demand is delivered or collected. Consequently, Cum-VRP prioritizes traversing the farthest arcs when the vehicle becomes lighter [21]. This method was later extended to incorporate customer arrivals as a cumulative component and minimize wait times [22]. The arrival time at a node was calculated as the total distance traveled to reach the current node, which was the sum of the arcs traversed to reach the point. This variation is known as the Cumulative Capacitated Vehicle Routing Problem (CCVRP), which has gained significant attention due to its applications in healthcare, disaster relief operations, maintenance, and customer-oriented logistics operations [9], [23].

Based on the description, this research aimed to develop a CCVRP model based on Time-Dependent factors in humanitarian logistics for disaster management. In this context, the minimum distance of the entire vehicle journey is defined as the travel duration between two nodes. In the Time-Dependent Vehicle Routing Problem (TDVRP), the vehicle speed or the time required to travel between two nodes depends on the journey's start time. The fastest route between the two points may become congested with vehicles upon arrival, potentially lengthening travel time. Therefore, the majority of previous research on CCVRP assumes that vehicle travel time remains constant regardless of future conditions [17], [24]. During periods with no road traffic, vehicle taking a longer route between two nodes may arrive at its destination more quickly [25].

To address the challenges, this research used CCVRP by incorporating Time-Dependent factors in humanitarian logistics for disaster management. Previous research on CCVRP has utilized various methods, including drones [26], time windows [27], [28], [29], and metaheuristics [28], [30], [31], [32], [33], [34], [35], [36], [37]. Recent advancements in the CCVRP include [28] developing a CUAVRP that optimizes vehicle route planning in humanitarian logistics scenarios using the GRASP-VND algorithm. This algorithm achieved efficient solutions by testing the Min-Sum objective on various instances with favorable results. Additionally, [29] heuristic and genetic algorithms were developed to enhance computational efficiency by optimizing the time complexity of the heuristic based on the CVRP. Another investigation [38] introduced the first Branch-and-Cut-and-Price algorithm, applying heuristic rules to balance useful routes in CCVRP with an optimal solution, demonstrating instance solvability to generate more routes and minimize arrival times. Despite these advancements, existing research still shows weaknesses, particularly in dynamically adapting to changing conditions such as road closures, diverse aid requests, and shifting priorities.

This research focuses on enhancing CCVRP by incorporating Time-Dependent factors, developing the existing CCVRP model, and applying humanitarian logistics delivery during natural disaster to optimize delivery times. This research aims to develop and evaluate a Cumulative Capacitated Vehicle Routing Problem with Time-dependent (CCVRP-TD) model to address the inefficiencies of traditional routing methods. The primary objective is to incorporate time-dependent variables into the model, reflecting real-world conditions that affect travel times and service efficiency, ultimately enhancing the responsiveness of humanitarian logistics operations. The results are expected to improve the efficiency of logistics delivery to disaster areas, thereby aiding government efforts in natural disaster management. The novelty of this research lies in the proposed Mixed Integer Nonlinear Programming (MINLP) model, which dynamically adapts to changing conditions such as road closures, diverse aid requests, and evolving priorities. The solution used for analysis combines exact methods with heuristic for computation, serving as model validation.

2. Methodology

2.1. Basic Concept of CCVRP-TD Analysis Model

In optimizing complex systems, there is a need to identify an optimal feasible integer region. This process includes several key steps to ensure the most effective solution is found while adhering to specific constraints. By systematically eliminating infeasible regions and refining potential solutions, the method guides the search toward the most promising areas, enhancing overall efficiency and effectiveness.

The procedure begins with identifying the feasible integer region and delineating potential solutions that satisfy the basic criteria. Subsequently, regions failing to meet the criteria are excluded, refining the search space. The next phase includes identifying areas capable of providing an optimal feasible solution, concentrating on regions with the highest potential to fulfill the objective function. This is followed by deriving gradient direction for the identified optimal area, guiding the search toward the most promising regions. The extent of movement along gradient direction is calculated, ensuring points remain within the feasible area. At this juncture, verification is conducted to ascertain when the point resides within an optimally feasible region to process concludes. However, when the point does not reside within the region, adjustments and refinements to the movement direction are implemented, followed by iteration to achieve an optimal feasible solution, as shown in figure 1.

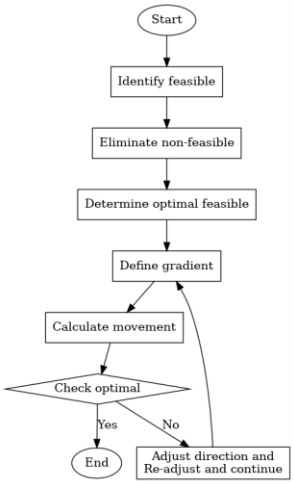

Figure 1. Optimization Process Flowchart for Feasible Solution Identification

The research starts with the identification of the objective function, which includes the selection of the best path for vehicles to meet consumer demand and minimize costs. The objective function that specifies the lowest possible travel expenses is given by expression (1). In this fundamental structure, the caterer's management focuses on reducing overall costs by making optimum use of the vehicles available for various types of deliveries. The overall price includes the expense of all vehicles used and secured throughout the daily planning horizon. The result of an improved CVRP-TD is shown in an example of model and optimization tests.

2.2. MINLP Model Framework

MINLP focuses on optimizing an objective function to satisfy a finite set of linear or nonlinear constraints and integrality conditions [39]. Recent research has provided solution to real-world problems such as water distribution system [40], weapon targeting in combat [41], cost-optimal and sustainable construction scheduling [42].

The optimization problem MINLP is [39] formulated to minimize the objective function $C^T X_2$, subject to a series of constraints that define the feasible solution space. The first constraint is a nonlinear bound, expressed as $\underline{g} \leq g\,(x) \leq \overline{g}$, where $g\,(x)$ represents a nonlinear function of the decision variables, constrained by lower and upper bounds g and $\overline{g}$, respectively. The second constraint is a linear inequality, $\underline{b} \leq A\,x \leq \overline{b}$, where A is a coefficient matrix defining the linear relationship between the decision variables, and b and $\overline{b}$ represent the lower and upper bounds for this relationship. Additionally, the decision variables x are subject to variable bounds, ensuring that they remain within the range $\underline{x} \leq x \leq \overline{x}$. Lastly, an integer constraint is applied, requiring that certain decision variables take values from the set of integers $x\tau \in Z^{|\tau|}$. Together, these constraints provide a structured framework for solving the optimization problem, ensuring that the solution adheres to all specified restrictions and conditions.

2.3. Description of the Problem Cum-VRP Time-Dependent Model

Incorporating time-dependent variables into the Cumulative Capacitated Vehicle Routing Problem with Time-dependent (CCVRP-TD) model is essential for accurately reflecting the dynamic conditions present during humanitarian logistics operations. In real-world scenarios, factors such as traffic congestion, road closures, and varying weather conditions significantly influence travel times and service durations [43]. For instance, during a disaster, access routes may become obstructed, leading to fluctuations in travel efficiency that traditional static models fail to account for. Research has demonstrated that neglecting these time variations can result in suboptimal routing decisions, ultimately hindering timely aid delivery. By integrating time-dependent variables, the CCVRP-TD model not only enhances the precision of routing solutions but also adapts to real-time conditions, improving logistical responsiveness. This approach aligns with recent advancements in the field, which emphasize the need for models that reflect the complexities of disaster management, thereby contributing to more effective humanitarian aid [44].

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The description of the problem can be expressed in a fully directed graph $G = (V, A)$, with $V = N \cup \{o\}$ a set of vertices and $A = \{(i,j) : i,j \in V\}$ a set of arcs. In this description, there is a binary variable representing the subscriber to the route and determining the sequential pair. The binary variables x_{ij}^r and y_i^r respectively define, when arc (i,j) and customer i belong to route r. Meanwhile, the binary variable z_{rs} determines when any vehicle traveling route r is followed by route s within weekdays. The notation $r < s$ indicates that the same vehicle is assigned to do route s after doing route r. The variable t_i^r represents the start time of service for customer i. When served by route r, and t_o^r and $t_o'^r$ represent the start and end times of route r, respectively, assumed M is a large enough number. The concise formulation for CVRP-TD is stated in the equation.

The following diagram may be used to illustrate the basic structure of CVRPTD. Let $G = (V, A)$ be a fully connected directed acyclic graph with vertex set to $V = \{0,1, \ldots, n\}$ and routes set to $A = \{(i,j): i,j \in V, i \neq j\}$. The distance (or trip cost) c_{ij} is specified for every possible path $(i,j) \in A$. The depot, known as vertex 0 ($i = 0$), is the hub from which the whole fleet operates. The consumers' vertices constitute the set defined by V_c. There is a daily demand $w_i \geq 0$ that is constant, a service time $s_i \geq 0$, and a service time window $[a_i, b_i]$ for each vertex. $i \in V_c$. Particularly, the demand $w = 0$ w = 0 and service time t = 0.

This presents a heterogeneous issue, where there are m distinct vehicle types in K vehicle fleet, each with capacity Q_m. Vehicle type m may make use of as many as n_m different vehicles. If K_m is a collection of m vehicle types, precisely one vehicle is assigned to each consumer. At the depot ($i = 0$), provide an arrival and departure timeframe for vehicles with $[a_0, b_0]$. A vehicle's arrival time at customer i is a_i and its departure time is b_i, while the set price for each type of vehicle is f_m. Additionally, each vehicle k in the route incurs a fixed purchase cost f_k. The central depot is the start and end point for each route, which must meet the time frame requirements. A vehicle cannot begin serving consumer i until after a_i and before b_i have completed their respective journeys. Specifically, a_i may be able to arrive before the vehicle does and wait for service.

Every consumer node $i \in V_c$ has a minimum service frequency F_i measured in days t per period, a daily demand W_i, and a service frequency σ^t. The node's daily demand is a factor in calculating the demand that has accrued between visits, denoted by the variable w_i. Since more goods pile up with less frequent service and more time is needed to load/unload at each stop, the cost of stopping at a node i, τ_i becomes a function of the schedule's periodicity. Each arc $(i,j) \in A$ has a predetermined travel expense (denoted by c_{ij}). In this research, there is a fleet of vehicles, K, with a certain capacity, C, showing the need to schedule operations over a period of time, T, which is the number of workdays.

The service benefit, denoted as δ^t in monetary terms, is associated with service selection and serves as an incentive to encourage more regular service delivery. In this approach, the rate of change in demand w_i is directly proportional to the increase in service benefits. The formalization of the variables used is outlined as follows: the binary variable x_{0j}^k indicates whether a vehicle of type $k \epsilon K$ delivers from the depot to customer $j \epsilon V_c$ (taking the value 1) or not (value 0). Similarly, the binary variable x_{ij}^m represents whether a vehicle of type $m \epsilon K_m$ serves the route between customers i and j ($i \neq j$) within V_c (value 1) or not (value 0). The variable z_0^m identifies whether a vehicle of type $m \epsilon K$ is available and active at the depot (value 1) or not (value 0). In addition, the continuous variable l_i^m represents the arrival time of a vehicle of type $m \in K_m$ at customer $i \in V_c$, expressed as a non-negative value. The service duration of a vehicle of type m at customer i is defined by u_i^m , also a non-negative continuous variable. Finally, Q_i is a cumulative variable representing the total quantity of goods delivered up to node i. This formalization provides a detailed quantitative framework for analyzing and optimizing vehicle-based logistics service operations.

2.4. CCVRP-TD Model

In developing the CCVRP-TD framework, several key assumptions are established to ensure the creation of a structured and feasible optimization model. These assumptions define the operational environment, set the constraints, and outline the objectives guiding the analysis and resolution of the problem. The framework assumes that all vehicles operate from a single central depot. The demand for each customer is considered constant, known in advance, and subject to increase cumulatively over time if the minimum service frequency is not met. Each customer is assigned a fixed service duration that remains consistent for every visit. Additionally, customers have specific time windows within which the service must commence. The vehicle fleet consists of heterogeneous vehicles, each characterized by distinct capacities and associated costs, with a maximum capacity for each vehicle that must not be exceeded during service. Finally, every customer has a minimum service frequency requirement that must be fulfilled within a specified period.

The mathematical model of the problem is expressed as follows.

$$\text{Minimize} \sum_{j\in V_c} C_{oj} \sum_{k\in K_c} x_{0j}^k + \sum_{(i,j)\in V_c} \sum_{m\in K_m} \sum_{t\in T} (\tau_{ij}^t\, \sigma_{ij}^t - w_{ij}^t \delta_{ij}^t) x_{ij}^t + \sum_{m\in K_m} f_m z_0^m \quad (1)$$

The mathematical model is structured to minimize the total cost of delivery operations while adhering to several operational constraints. The objective function (Equation 1) is designed to minimize the total cost, which comprises three key components: (1) the transportation cost $\sum_{j\in V_c} C_{oj} \sum_{k\in K_c} x_{0j}^k$, representing the cost of starting routes from the depot to customers; (2) the dynamic cost $\sum_{(i,j)\in V_c} \sum_{m\in K_m} \sum_{t\in T} (\tau_{ij}^t\, \sigma_{ij}^t - w_{ij}^t \delta_{ij}^t) x_{ij}^t$, which accounts for the time, distance, and customer-specific service benefits; and (3) the fixed cost $\sum_{m\in K_m} f_m z_0^m$, associated with activating vehicles at the depot. This objective captures the complexity of balancing cost efficiency, dynamic routing, and customer satisfaction in delivery operations.

$$\sum_{k\in K_c} x_{0j}^k = 1,\ \forall j \in V_c \quad (2)$$

The model is subject to several constraints to ensure operational feasibility. Equation (2) ensures that every customer j is served exactly once by summing over all vehicles k that may visit the customer. This guarantees complete service coverage across all customers.

$$\sum_{k\in K} \sum_{j\in V} x_{ij}^k = \ \forall i \in V_c \quad (3)$$

Equation (3) enforces that each vehicle k starts and ends its route at the depot, maintaining the route's continuity.

$$\sum_{i\in V} x_{ij}^k - \sum_{i\in V} x_{ij}^k = \ ;\ \ \forall j \in V_c, \forall k \in K \quad (4)$$

Additionally, Equation (4) introduces a flow conservation constraint, ensuring that for every node j, the number of vehicles entering a node is equal to the number leaving it.

$$x_{ij}^k \le z_0^m, (i,j) \in V_c, \forall m \in K_m \quad (5)$$

Equation (5) links the routing decisions to vehicle activation, ensuring that a route x_{ij}^k is feasible only if the corresponding vehicle k is active ($z_0^k = 1$).

$$\sum_{j\in V_c} x_{ij}^k \le 1; \forall k \in K \quad (6)$$

$$\sum_{j\in V_c, i>1} x_{ij}^k \le 1; \forall k \in K \quad (7)$$

Equation (6) limits each vehicle k to a single trip, preventing its overutilization. Equation (7) further restricts that no more than one route can be assigned to a vehicle at any given time.

$$\sum_{i\in V_c} d_i \sum_{j\in V_c} x_{ij}^m \le Q_m \quad (8)$$

$$\sum_{i=1}^m q_i \le Q \quad (9)$$

Capacity constraints are critical in logistics optimization. Equation (8) ensures that the total demand served by any vehicle m does not exceed its capacity Q_m. Similarly, Equation (9) ensures that the total cumulative load q_i across all vehicles does not surpass the overall fleet capacity Q. These constraints are crucial for balancing load distribution across the fleet and avoiding overloading.

$$x_{ij}^m \left(l_i^m + u_i^m + s_i + t_{ij} - l_j^m \right) = 0; \forall \in K_m, (i,j) \in A \quad (10)$$

$$l_i^m \le a_i \sum_{j\in V_c} x_{ij}^m; \forall m \in K_m\ , i \in V_c \quad (11)$$

Time-based constraints ensure that the scheduling and delivery times are adhered to. Equation (10) enforces consistency in the travel times, service durations, and arrival times. This equation links the arrival time l_i^m, service duration u_i^m, and travel time t_{ij}, ensuring that delivery operations align with the planned schedule. Equation (11) ensures that the arrival times are within the predefined service time windows, thus meeting customer requirements.

$$a_i \sum_{j\in V_c} x_{ij}^m \leq l_i^m u_i^m \leq b_i \sum_{j\in V_c} x_{ij}^m; \forall m \in K_m \ , i \in V_c \quad (12)$$

$$\sum_{j\in V_c} w_j x_{oj}^m \leq n_m ; \forall m \in K_m \quad (13)$$

Logical and feasibility constraints are incorporated to maintain consistency in decision variables. Equation (12) specifies bounds for arrival times l_i^m and service durations u_i^m within their allowable ranges. Equation (13) ensures that binary variables x_{ij}^m , x_{oj}^m , $z_o^m \in \{0,1\}$represent valid decisions for routing and vehicle activation.

$$x_{oj}^m, x_{ij}^m, z_o^m \in \{0,1\}; \quad \forall i \in V, \forall j \in V_c, \forall k \in K, \forall m \in K_m \quad (14)$$

$$l_i^m, u_i^m \geq 0 ; \forall i \in V_c, \forall m \in K_m \quad (15)$$

Finally, Equations (14) and (15) impose non-negativity constraints on continuous variables l_i^m, u_i^m, ensuring realistic values for times and service durations. This detailed formulation integrates dynamic routing, vehicle capacity, and time constraints while optimizing costs. The model's novelty lies in its ability to incorporate time-dependent costs, service benefits, and multi-depot coordination in a unified framework, making it highly adaptable for modern logistics challenges.

3. Results And Discussion

3.1. Problem Definition and Assumptions

The CCVRP-TD framework for efficient logistics involves planning the combined routing of four delivery vehicles to serve six customers across five distinct routes. This problem incorporates the cumulative metrics of the entire fleet, considering multiple supplier depots and constrained yet flexible delivery time windows. The problem details specify the use of four vehicles to serve six customers across five routes, with operations involving multiple suppliers and depots. Deliveries are required to adhere to flexible but constrained time windows. Several key assumptions underlie this framework: each vehicle begins and ends its route at a depot; the primary objective is to minimize the cumulative travel distance or time for all vehicles; every customer must be visited within their designated time window; and all vehicles operate within a limited capacity that cannot be exceeded.

The method for solving the CCVRP-TD includes using MINLP optimization to achieve the optimal routing and scheduling of the delivery vehicles. It also focuses on minimizing the cumulative travel distance or time across all vehicles while ensuring that each consumer is visited within their specified time window and maintaining vehicle capacity constraints.

3.2. The Proposed Method to Solve CCVRP-TD

The iterative method involves several key steps to produce a viable descent direction, denoted as p. First, the reduced gradient gA is obtained using the equation gA = Z gT. Next, the Hessian reduction is approximated, expressed as $G_A \doteqdot Z^T GZ$. The third step calculates the solution for the system of equations Z GZpTA = Z g–T, which is simplified by breaking the system G pAA =– gA. Afterward, the search direction p is determined using p = ZpA. Finally, a row search is conducted to find an approximation for α^*, which satisfies $f(x + \alpha^* p) = \min_{\substack{\alpha \\ \{x+\alpha p \text{ feasibel}\}}} f(x + \alpha p)$. These steps ensure an effective and systematic approach to descent direction optimization.

For example, Z is not limited to one shape since it is the only restriction on Z (algebraically) and has a complete column rank. The form of Z that represents the actual operation is as follows:

$$Z = \begin{bmatrix} -W \\ I \\ 0 \end{bmatrix} = \begin{bmatrix} -b^{-1}S \\ I \\ 0 \end{bmatrix} \begin{matrix} \}m \\ \}s \\ \}n-m-s \end{matrix} \quad (16)$$

This simple representation is used for computing purposes with S and triangular (LU) factorizations of B, but not for calculating the Z matrix. Based on the preceding discussion of steps A through D in equation [B S], the fundamental benefit of the Z transformation is that it does not bring extra conditioning into the minimization issue. This method has been included in the code when Z is expressly kept as a dense matrix. The LDV factorization of the [BS] matrix allows for the extension to a linear constraint with a sparse distribution that is specified in advance.

[B S] = [L O]DV

Using the product form of L and V to store the triangle (L), diagonal (D), and orthogonal ($D^{1/2}V$). This factorization is often denser than the LU factorization of B, but only when S contains more than 1 or 2 columns. Therefore, the continuous use of Z in (5.3) was proposed for expediency.

3.3. CCVRP-TD Model Procedure and Algorithm

The optimization procedure is briefly described based on the following assumptions. An eligible vector x satisfies the condition [B S N]x = b, with l≤ x≤ u. The corresponding function value f(x) and gradient vector g(x) = [gB gs g]NT are determined. The number of superbase variables s lies within the range 0≤ s≤ n– m. Factorization is performed using the LU decomposition on the matrix B of size m× m. Additionally, the quasi-Newton approximation to the s × s matrix is factored as ZT GZ, although G, Z, and ZT GZ are never explicitly calculated. A vector π is then obtained such that BTπ = gB. The reduced gradient vector is computed as h = gs - STπ. Finally, small positive convergence tolerances, denoted as TOLRG and TOLD, are introduced to ensure accuracy and consistency during the optimization process.

3.4. The CCVRP-TD Algorithm Model

The optimization process consists of two stages, with detailed steps for each. In Stage 1, the process begins by identifying a row i* that contains a basic non-feasible solution, ensuring that $\delta_{i*} = \min\{f_i, 1 - f_i\}$. Next, a pricing operation is performed, denoted as $v_{i*}^T = e_{i*}^T B^{-1}$, where the reduced costs of the nonbasic variables are calculated during this column selection. The nonbasic variable j is then moved from its boundary by determining the maximum allowable movement $\sigma_{ij} = V_{i*}^{\tau}\alpha_j$, adjusting column j* by increasing it from the lower bound (LB) or decreasing it from the upper bound (UB). If this adjustment is not possible, the process moves to the next i*. Subsequently, the matrix B is updated as B =αj* αj* for αj*, followed by performing ratio tests on the basic variables to ensure feasibility while adjusting the nonbasic j* from its bounds. After exchanging the basis, if row i* becomes empty, the process transitions to Stage 2; otherwise, the steps are repeated from Step 2. In Stage 2, the focus shifts to achieving integer feasibility. This involves adjusting infeasible superbases using fractional steps to attain complete integer feasibility. Additionally, the integer feasible superbase is further refined through a neighborhood search, ensuring local optimality.

3.5. Cumulative Result Analysis

The cumulative result analysis focuses on evaluating the combined performance of the entire fleet of delivery vehicles in terms of routing efficiency, total travel distance, and adherence to delivery time windows. This analysis provides a comprehensive assessment of logistical effectiveness by considering multiple factors, including vehicle capacity, customer demand, and route optimization within flexible but constrained delivery time frames. Using Mathematical Programming System (MPS) software, advanced MINLP optimization was applied to derive these results. The overall efficiency and effectiveness of the proposed routing solution were determined based on cumulative metrics.

The routes assigned to each vehicle are as follows: Vehicle 1 covered the route Depot A → Customer 1 → Customer 4 → Depot B; Vehicle 2 followed Depot A → Customer 2 → Customer 5 → Depot C; Vehicle 3 operated from Depot B → Customer 3 → Customer 6 → Depot A; and Vehicle 4 traveled from Depot C → Customer 1 → Customer 5 → Depot A. The cumulative distance traveled by each vehicle was 25 km for Vehicle 1, 30 km for Vehicle 2, 20 km for Vehicle 3, and 35 km for Vehicle 4, resulting in a total cumulative distance of 110 km.

The delivery schedule adhered strictly to specific time windows: Customer 1 (9:00 AM - 11:00 AM), Customer 2 (10:00 AM - 12:00 PM), Customer 3 (11:00 AM - 1:00 PM), Customer 4 (12:00 PM - 2:00 PM), Customer 5 (1:00 PM - 3:00 PM), and Customer 6 (2:00 PM - 4:00 PM). The vehicles delivered as follows: Vehicle 1 served Customer 1 at 9:30 AM and Customer 4 at 10:30 AM; Vehicle 2 served Customer 2 at 10:30 AM and Customer 5 at 11:30 AM;

Vehicle 3 served Customer 3 at 11:30 AM and Customer 6 at 12:30 PM; and Vehicle 4 served Customer 1 at 9:45 AM and Customer 5 at 11:00 AM. This structured approach highlights the efficiency and accuracy of the proposed routing system, ensuring timely deliveries while minimizing total travel distance.

The cumulative result provides an optimized routing plan that balances the delivery time windows, vehicle capacity, and travel distance. By coordinating multiple depots and suppliers, the distribution process is made more efficient, reducing the total travel distance and time cumulatively. Furthermore, the flexible constrained time windows are managed effectively to ensure customer satisfaction and operational efficiency. In the context of disaster management, these optimized routing strategies are crucial for ensuring timely and effective delivery of humanitarian aid. During disaster, efficient logistics can significantly impact the survival and well-being of affected populations. By minimizing the total travel distance and optimizing delivery schedules, resources such as food, medical supplies, and emergency equipment can be delivered quickly and reliably. The optimization also enhances smooth delivery under challenging conditions such as road blockages or varying levels of urgency in aid requests.

The optimized routing plan leads to a significant reduction in total cumulative travel distance, as shown by the assignment of specific routes to each vehicle and the detailed scheduling of delivery. For instance, Vehicles 1, 2, 3, and 4 travel a total of 25 km, 30 km, 20 km, and 35 km, respectively, resulting in a total cumulative distance of 110 km. Each vehicle adheres to the specified time windows for customer deliveries, ensuring completion within the allocated time frames. The scheduling efficiency is further enhanced by considering the cumulative delivery schedule, which ensures that each customer is visited at the optimal time. For example, Customer 1 is served at 9:30 AM and 9:45 AM by Vehicles 1 and 4, respectively. Meanwhile, Customer 5 is served at 11:00 AM and 11:30 AM by Vehicles 2 and 4, respectively. This careful coordination minimizes waiting times and maximizes the use of available vehicle capacity. The vehicle routing diagram details the sequence of deliveries made by each vehicle, underscoring the systematic approach to aid distribution during emergencies (figure 2)

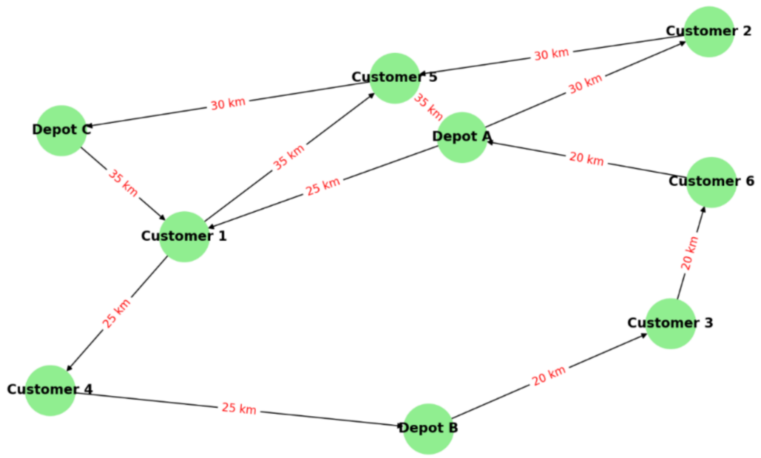

Figure 2. Vehicle Routing Diagram for Optimized Aid Distribution

In disaster scenarios, the ability to adapt routing plans dynamically based on real-time conditions is essential to maintain the effectiveness of disaster response efforts. The use of MINLP optimization allows for adjustments to be made in response to changing conditions, such as road closures or shifts in demand, ensuring quick delivery. In conclusion, the cumulative result analysis, facilitated by the use of MPS software and MINLP optimization, shows an efficient method of vehicle routing that is highly applicable to disaster management. The analysis shows the importance of considering multiple factors, such as road conditions, vehicle capacity, and delivery time windows, to achieve optimal routing solutions. These factors can enhance logistical efficiency and ensure timely delivery of humanitarian aid during emergencies.

4. Conclusion

In conclusion, this research showed significant advancements in solving the CCVRP-TD using MPS software. This method proficiently addressed disaster logistics optimization challenges, yielding optimal solutions. By integrating

MINLP model with a hybrid method, the research successfully incorporated capacity and time dependencies through rigorous analytical and exact proofs. This integration identified key variables that determined the fastest routes, thereby achieving optimal solutions. The introduction of a novel model for Cum-VRP problems, accounting for both capacity and time dependencies, offered a significant contribution. The results provided valuable information for further advancements through the integration of Artificial Intelligence methods, such as Deep Learning and Machine Learning, which could enhance human efforts across various applications. This research also contributed to the field of Computer Science by offering innovative perspectives and methods for solving complex routing problems. The research showed an optimized Cum-VRP solution for efficient logistics, effectively planning the routing for four delivery vehicles serving six customers. It managed multiple depots and flexible delivery time windows, ensuring timely delivery, minimal travel distances, and enhanced overall logistics efficiency. This comprehensive method provided a holistic view of the distribution network's performance, showing the practical implications of the results.

5. Declarations

5.1. Author Contributions

Conceptualization: D.H., W., and I.S.D.; Methodology: W.; Software: D.H.; Validation: D.H., W., and I.S.D.; Formal Analysis: D.H., W., and I.S.D.; Investigation: D.H.; Resources: W.; Data Curation: W.; Writing Original Draft Preparation: D.H., W., and I.S.D.; Writing Review and Editing: W., D.H., and I.S.D.; Visualization: D.H. All authors have read and agreed to the published version of the manuscript.

5.2. Data Availability Statement

The data presented in this study are available on request from the corresponding author.

5.3. Funding

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5.4. Institutional Review Board Statement

Not applicable.

5.5. Informed Consent Statement

Not applicable.

5.6. Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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