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# Developing a Parallel Network Slack-Based Measure Model in the Occurrence of Hybrid Integer-Valued Data and Uncontrollable Factors

Syarifah Nurfuaduz Zakiah Habib Dzulkarnain[1,*], Mohd Kamal Mohd Nawawi[2], Rosmaini Kashim[3]

[1,2,3]*School of Quantitative Sciences, Universiti Utara Malaysia, 06010 UUM Sintok, Kedah, Malaysia*

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**Abstract**

This study develops an alternative approach to the parallel network Slack-Based Measure (SBM) Data Envelopment Analysis (DEA) model, offering a more accurate and informative assessment of performance within a network system. Traditional DEA models solely focus on the input utilization and the outputs produced when assessing efficiency, disregarding the operation of internal processes within a network system. In addition, these approaches do not assess the concurrent requirement of hybrid integer-valued data and uncontrollable factors on efficiency measures. To address these gaps, we propose a novel approach to parallel network SBM DEA model that integrates hybrid integer-valued data with uncontrollable factors, aiming for a more precise evaluation. Both requirements were initially integrating into the existing method. Subsequently, the optimal solution for the proposed method was achieved by converting its fractional form into a linear one. Therefore, the measures of the proposed approach can now deal directly with controllable hybrid integer- valued input and output slacks. We applied this model to a dataset of 26 faculties in a Malaysian public university, followed by a comparative analysis with existing models. Empirical findings indicate that four (4) faculties are found to be overall effective, as all of their internal processes are effective, while the other faculties are ineffective since not all of their internal processes are effective. The results from our model enable decision-makers to identify ineffectiveness within network processes, thereby facilitating targeted improvements in system performance. By concentrating on the appropriate processes, management can enhance their overall effectiveness and internal effectiveness.

*Keywords:* Parallel, Data Envelopment Analysis, Uncontrollable Factors, Slack-Based Measure, Integer-Valued Data

## 1. Introduction

Data Envelopment Analysis (DEA) refers to a non-parametric technique employed to assess the relative efficiency of a similar set of Decision-Making Units (DMUs) utilizing multiple inputs to generate multiple outputs. In Traditional DEA models, the efficiency assessment is influenced by the underlying technological assumptions of Constant Returns to Scale (CRS) and Variable Returns to Scale (VRS). The CRS model, developed by Charnes, Cooper, and Rhodes (namely as the CCR model), assumes that the relationship between the efficiency and scale of operations (technology) remains constant [1]. In contrast, the VRS model, developed by Banker, Cooper, and Charnes (namely as the BCC model), considers the variability in the operation's scale [2]. Differing from the CCR model, the BCC model considers that the DMU's efficiency can vary based on whether it operates in the region of CRS, Decreasing Returns to Scale (DRS), or Increasing Returns to Scale (IRS).

These traditional DEA models treat each DMU as a "black box," assessing efficiency solely based on transforming multiple inputs into multiple outputs. This model disregards the operation of internal processes in a DMU's system [3], leading to inaccuracies in efficiency scores [4]. As a result, a DMU may have a high-efficiency score even if its internal processes are inefficient. To overcome this limitation, Färe and Grosskopf [5] introduced the first network DEA model to investigate the internal processes and examine the network system's efficiency. Consequently, researchers have undertaken various efforts to decompose system efficiency into processes to discriminate against inefficient DMUs.

Network DEA has become increasingly popular, and the two-stage network DEA model has gained substantial attention in recent studies. However, recognizing that real-world organizations often comprise parallel production

***Corresponding author: Syarifah Nurfuaduz Zakiah Habib Dzulkarnain (syazakiah@gmail.com)**
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processes, Kao [6] introduced a parallel DEA model that simultaneously assesses system as well as process efficiencies in a single linear program. Building upon this foundation, subsequent studies [7], [8], [9] have further developed the DEA models for assessing the efficiency of parallel production systems.

All the aforementioned network DEA models utilize radial efficiency measures, which assume proportional changes in inputs and outputs [10]. However, radial models do not consider inputs and outputs as slack variables [11]. To address this issue, a non-radial network Slack Based Measure (SBM) model was introduced [12]. This model simultaneously accounts for input and output slacks without assuming proportional changes, setting non-radial models apart from radial ones. While many studies have applied the network SBM model to efficiency assessments [13], [14], [15], these models generally assume non-integer-valued data and fully controllable inputs and outputs. However, real-world scenarios often involve integer-valued and uncontrollable factors, necessitating more accurate modeling approaches.

In real managerial situations, inputs and/or outputs may naturally take on integer values (e.g., the number of patients, rooms, or students). Rounding the non-integer values of inputs and/or outputs to their nearest integer values can lead to inaccuracies in efficiency scores [16]. Therefore, previous research studies have introduced an integer-valued DEA model to address these limitations [17], [18], [19]. However, these studies have primarily focused on radial integer DEA models and do not account for internal processes, leading to system inefficiencies in DMUs. Relying on the fundamental principles of integer-valued requirements and the non-radial network DEA model, Ajirlo et al. [20] developed a two-stage network slack-based integer-valued DEA model. The model can discriminate between internal process inefficiencies concerning integer-valued data and provide the decision-maker with information on improving the internal process's inefficiency.

The integer-valued DEA models assume complete managerial controls over all inputs and outputs. Charnes and Cooper [21] introduced the concept of uncontrollable factors in DEA to overcome the issue of inputs or outputs lying beyond managerial control. Few studies have concurrently incorporated integer-valued DEA and uncontrollable factors. For example, Taleb et al. [22] introduced an approach to a super-efficient SBM DEA model incorporating integer-valued and uncontrollable factors. In the context of the network DEA, Abdali and Fallahnejad [23] developed a two-stage model incorporating uncontrollable inputs. However, these approaches do not consider both integer-valued and uncontrollable factors within an SBM framework for network systems.

In this study, we introduced an alternative parallel SBM DEA model considering hybrid integer-valued data (measure both integer and non-integer-valued data and uncontrollable factors. Two contributions of our alternative approach to DEA should be highlighted: (1) the model possesses the capability to discriminate between inefficient and efficient DMUs by considering the operations of internal processes in a parallel network system, and (2) the model addresses the scenario where certain inputs and/or outputs are hybrid integer-valued and uncontrollable factors. This consideration sets our approach apart from previous studies that have not explored this combination of factors in the network systems. With these salient features, this model produces a more accurate assessment than those attained from the traditional black-box technique in the occurrence of hybrid integer-valued and uncontrollable factors.

The remainder of this study is organized as follows: Section 2 discusses the foundational frameworks of existing DEA models, leading to our proposed alternative approach. Section 3 introduces our approach, which incorporates hybrid integer-valued data and uncontrollable factors. Section 4 provides a numerical example using a dataset from a Malaysian public university. Finally, Section 5 concludes the study.

## 2. Preliminaries

The first SBM DEA model, developed by Tone [11], considers the existence of slacks, allowing for a more accurate assessment of DMUs' efficiency. Consider a set of $j$ homogeneous DMUs. Each DMU utilizes a set of $h$ inputs to generate a set of $m$ outputs. The individual units are denoted by DMU $k$, where $k$ ranges from 1 to $j$. Let $x_{rk}$ and $y_{sk}$ represent the $r^{th}$ input and $s^{th}$ output of the $k^{th}$ DMU, respectively. The positive weight of the efficient DMU under assessment, $DMU_O$ residing on the efficient frontier is denoted by $\lambda_k$. Therefore, to compute the efficiency score of $DMU_O$, the SBM model is formulated as:

$$p_o = min. \frac{1 + \frac{1}{h}\sum_{r=1}^{h} \frac{t_{r_o}^-}{x_{ro}}}{1 - \frac{1}{m}\sum_{s=1}^{m} \frac{t_{s_o}^+}{y_{so}}}, \quad (1)$$

subject to:

$\sum_{k=1}^{K} \lambda_k X_{rk} + t_{r_o}^- = x_{ro}, r = 1, \ldots, h,$

$\sum_{k=1}^{K} \lambda_k Y_{sk} - t_{s_o}^+ = y_{so}, s = 1, \ldots, m,$

$\lambda_k, t_{r_o}^-, t_{s_o}^+ \geq 0$, k$= 1, \ldots, K.$

Definition 1 (SBM-efficient). A DMU $(x_o, y_o)$ is considered SBM-efficient if score of $p$*=1[11].

However, when a network system is present, model (1) may not accurately measure efficiency, where the efficiency of one DMU is intricately linked to the operations of internal processes. Therefore, in the following discussion, we will offer a precise overview of a parallel network SBM DEA model by Kao in 2020 [24]. Subsequently, we will utilize insights from Kao's model as a foundation to propose an alternative model.

**Kao's parallel SBM DEA Model**

The structure of general parallel system with $p$ processes is depicted in figure 1, in which each process $q$ utilizes the inputs $X_r^{(q)}$ , $r = h^{(q-1)}, \ldots, h^{(q)}$ to produce the outputs $X_s^{(q)}$, $s = m^{(q-1)} + 1, \ldots, m^{(q)}$, with $h^{(0)} = 0$, $h^{(p)} = h, m^{(0)} = 0$ $and$ $m^{(p)} = m$. The processes operate independently to produce outputs from inputs, and there are no intermediate products.

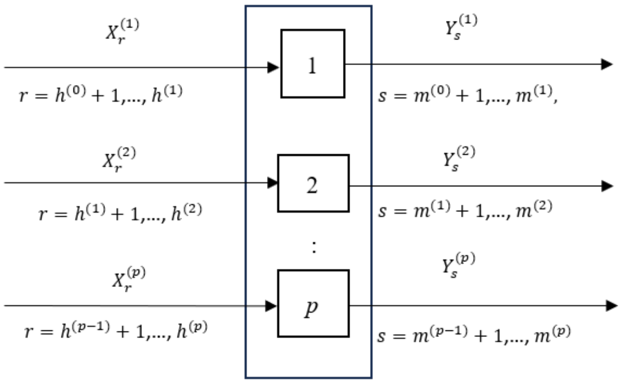


**Figure 1.** The structure of the parallel system, Kao [24]

Additionally, Tone and Tsutsui [25] defined some properties of a network SBM DEA model as follows:

Definition 2. A DMU is overall efficient if and only if it is efficient for all processes.

Definition 3. Every DMU possesses at least one efficient process.

Traditional DEA models assume that inputs and/or outputs are non-integer values. However, when dealing with categorical or ordinal data [26], [27], there is a need to handle integer-valued data in DEA. Lozano and Villa [28] pioneered an integer radial DEA model, assuming all inputs or outputs are integer values. Nevertheless, this is a significant limitation: it violates key DEA principles such as free disposability, convexity, and CRS [2]. To address this limitation, Kuosmanen and Matin [29] introduced new axiom foundations for integer-valued DEA models, introducing a new Production Possibility Set (PPS) that accommodates both non-integer and integer inputs and outputs within the BCC model. This approach, known as a hybrid integer-valued DEA, includes a hybrid integer linear programming equation for assessing the efficiency of input-oriented DEA models. Subsequently, they expanded the model to integrate considerations of return to scale, encompassing scenarios of non-increasing, constant, or non-decreasing scale nature [30]. Notably, the model avoids overestimating efficiency measures investigated in Lozano and Villa's model [28] by dealing with hybrid integer-valued inputs and outputs. Nevertheless, these proposed models were primarily developed under the radial framework of input- and output-oriented models, which do not account for slack in inputs and outputs simultaneously.

Past researchers have explored non-radial integer-valued DEA models as alternatives to the radial model, drawing from the foundational principles outlined by Matin and Kuosmanen [30]. For instance, Matin [31] presented a novel concept, incorporating integer-valued data into the additive model initially proposed by Banker et al. [2]. An approach for achieving super efficiency in an additive integer-valued DEA model has also been introduced, which involves a two-stage process to categorize DMUs as either inefficient or efficient [32]. Subsequently, efficient DMUs were ranked based on non-integer and integer input and output values. This method was later expanded by Chen et al. [33] to accommodate integer-valued outputs. To date, a range of radial and non-radial models have been introduced in the presence of integer-valued data [16]. However, a common assumption in these integer-valued DEA models is that the management of a DMU has complete control over all inputs and outputs.

In real managerial scenarios, certain inputs and/or outputs are not controlled by the management of a DMU. These elements are referred to as uncontrollable factors and hold a substantial role in the production system's processes. Ignoring these uncontrollable factors when assessing the DMU's performance leads to inaccurate efficiency scores. Some examples of uncontrollable inputs and/or output factors are student enrolment [34], employment rates [35], and the number of hospitalizations [36]. To ensure accurate efficiency assessments, the constraints associated with the uncontrollable input and/or output must be modified to explicitly reject any decrease in inputs or increment in outputs.

Consequently, the objective function should focus solely on controllable inputs and outputs, as their levels are not within the control of DMU management. By excluding uncontrollable inputs and outputs from the objective function, the efficiency assessment can provide valuable insights into the strategic decisions made by the DMU in managing factors under its control. Numerous studies have proposed DEA models that account for uncontrollable factors [37], [38], [39], [40], [41], [42]. However, these studies are predominantly grounded in the principles of the radial model, which has the limitation of not simultaneously addressing both input and output slacks.

The initial non-radial SBM model that accounts for the uncontrollable factors was developed by Farzipoor Sean [43]. Building on this, Esmaeili [44] advanced the SBM model to handle uncontrollable factors by focusing its objective function solely on controllable inputs and outputs, thereby excluding the slack associated with uncontrollable inputs and outputs from the relevant constraints. The resulting efficiency assessments are more accurate and less than those obtained from Banker and Morey's [26] model, highlighting the significance of concurrently addressing the slack of inputs. For an extensive understanding of efficiency assessment, Hua et al. [45] developed the SBM model in the presence of uncontrollable input and undesirable output. Furthermore, other researchers have also expanded the SBM model, considering a more collective range of factors, including controllable, uncontrollable, undesirable, and desirable factors [46]. Additionally, a super-efficiency SBM model was introduced in the event of both hybrid integer-valued and uncontrollable factors, further enriching the accuracy of models in capturing the diversity of real managerial scenarios [22]. These collective efforts show DEA models non-integer evolution to address multifaceted factors in assessing efficiency.

However, while these studies have fundamentally focused on a black-box (single) DEA model, a notable shift has been towards a more accurate and informative approach [47]. Specifically, to internally discriminate the DMU in a network system in the occurrence of uncontrollable factors. For instance, Abdali and Fallahnejad [23] developed a network two-stage DEA approach to provide a more inclusive analysis, allowing for a detailed efficiency assessment in the network system of DMUs in the occurrence of uncontrollable factors.

To further enhance the assessment, this study aims to develop a parallel SBM DEA model that concurrently addresses hybrid integer-valued data and uncontrollable factors.

## 3. The Proposed Methodology

Recall Kao's parallel SBM DEA approach, which deals with input and output slacks and can capture internal efficiency. However, the approach assumed that all inputs and outputs are non-integer values controlled by a DMU's management. Therefore, to extend the DEA model to diverse real managerial scenarios, this study introduced an alternative approach incorporating hybrid integer values and uncontrollable factors into the model.

To formulate the mathematical expressions for the proposed model, consider classifying inputs and outputs into two distinct categories: non-integer values and integer values in the occurrence of controllable and uncontrollable factors, i.e.

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$$r_1 \in r_1^C \cup r_1^{UC}, r_1^C \cap r_1^{UC} = \Phi,$$

$$r_2 \in r_2^C \cup r_2^{UC}, r_2^C \cap r_2^{UC} = \Phi,$$

$$s_1 \in s_1^C \cup s_1^{UC}, s_1^C \cap s_1^{UC} = \Phi, \qquad s_2 \in s_2^C \cup s_2^{UC}, s_2^C \cap s_2^{UC} = \Phi,$$

where $r_1^C$ and $s_1^C$ represent the controllable non-integer inputs-outputs, $r_1^{UC}$ and $s_1^{UC}$ refer to the uncontrollable non-integer inputs-outputs, $r_2^C$ and $s_2^C$ signify the controllable integer inputs-outputs while $r_2^{UC}$ and $s_2^{UC}$ represent the uncontrollable integer outputs and inputs of $DMU_j$, accordingly.

A parallel system with $p$ process, in which each process $q$ utilizes the inputs $X_r^{(q)}$ , $r = R^{(q-1)} + 1, \dots R^{(q)}$ in producing the outputs $Y_s^{(q)}$, $s = S^{(q-1)} + 1, \dots, S^{(q)}$, with $R^{(0)} = 0, R^{(P)} = \mathrm{R}, S^{(0)} = 0, S^{(p)} = \mathrm{S}$. The proposed approach of hybrid integer-valued and uncontrollable factors into the parallel SBM DEA model is formulated as follows:

$$\rho_{\mathrm{HIC}}^{\mathrm{pSBM}} = \min. \frac{1 + \frac{1}{R_1^{C(q)} + R_2^{C(q)}} \sum_{q=1}^{p} \left( \sum_{r_1^C = R_1^{C(q-1)}+1}^{R_1^{C(q)}} \frac{t_{r_1 c_o}^{(q)-}}{x_{r_1 c_o}^{(q)-}} + \sum_{r_2^C = R_2^{C(q-1)}+1}^{R_2^{C(q)}} \frac{t_{r_2 c_o}^{(q)-}}{x_{r_2 c_o}^{(q)}} \right)}{1 - \frac{1}{S_1^{C(q)} + S_2^{C(q)}} \sum_{q=1}^{p} \left( \sum_{s_1^C = S_1^{C(q-1)}+1}^{S_1^{C(q)}} \frac{t_{s_1 c_o}^{(q)+}}{y_{s_1 c_o}^{(q)}} + \sum_{s_2^C = S_2^{C(q-1)}+1}^{S_2^{C(q)}} \frac{t_{s_2 c_o}^{(q)+}}{y_{s_2 c_o}^{(q)}} \right)} \tag{2}$$

subject to:

$$\sum_{k=1}^{K} \lambda_k^{(q)} x_{r_1 c_j}^{(q)} + t_{r_1 c_o}^{(q)-} = x_{r_1 c_o}^{(q)}, r_1^C \in R_1^{C(q)}, r_1^C = R_1^{C(q-1)} + 1, \dots, R_1^{C(q)},$$

$$\sum_{k=1}^{K} \lambda_k^{(q)} x_{r_2 c_j}^{(q)} + t_{r_2 c_o}^{(q)-} = x_{r_2 c_o}^{(q)}, r_2^C \in R_2^{C(q)}, r_2^C = R_2^{C(q-1)} + 1, \dots, R_2^{C(q)},$$

$$\sum_{k=1}^{K} \lambda_k^{(q)} x_{r_1 UC_j}^{(q)} = x_{r_1 UC_O}^{(q)}, r_1^{UC} \in R_1^{UC(q)}, r_1^{UC} = R_1^{UC(q-1)} + 1, \dots, R_1^{UC(q)},$$

$$\sum_{k=1}^{K} \lambda_k^{(q)} x_{r_2 UC_j}^{(q)} = x_{r_2 UC_O}^{(q)}, r_2^{UC} \in R_2^{UC(q)}, r_2^{UC} = R_2^{UC(q-1)} + 1, \dots, R_2^{UC(q)},$$

$$\sum_{k=1}^{K} \lambda_k^{(q)} y_{s_1 c_j}^{(q)} - t_{s_1 c_o}^{(q)+} = y_{s_1 c_o}^{(q)}, s_1^C \in S_1^{C(q)}, s_1^C = S_1^{C(q-1)} + 1, \dots, S_1^{C(q)},$$

$$\sum_{k=1}^{K} \lambda_k^{(q)} y_{s_2 c_j}^{(q)} - t_{s_2 c_o}^{(q)+} = y_{s_2 D_o}^{(q)}, s_2^C \in S_2^{C(q)}, s_2^C = S_2^{C(q-1)} + 1, \dots, S_2^{C(q)},$$

$$\sum_{k=1}^{K} \lambda_k^{(q)} y_{s_1 UC_j}^{(q)} = y_{s_1 UC_O}^{(q)}, s_1^{UC} \in S_1^{UC(q)}, s_1^{UC} = S_1^{UC(q-1)} + 1, \dots, S_1^{UC(q)},$$

$$\sum_{k=1}^{K} \lambda_k^{(q)} y_{s_2 UC_j}^{(q)} = y_{s_2 UC_O}^{(q)}, s_2^{UC} \in S_2^{UC(q)}, s_2^{UC} = S_2^{UC(q-1)} + 1, \dots, S_2^{UC(q)},$$

$$\sum_{k=1}^{K} \lambda_k^{(q)} = 1,$$

$$\lambda_k^{(q)}, x_{r_1 c_o}^{(q)}, y_{s_1 c_o}^{(q)} \geq 0, \mathrm{k} = 1, \dots, K$$

$$t_{r_1 c_o}^{(q)-}, t_{s_1 c_o}^{(q)+} \geq 0, \mathrm{q} = 1, \dots, p$$

$$x_{r_2 c_o}^{(q)} \in Z_+^{R_2^{C\,(q)}}, y_{s_2 c_o}^{(q)} \in Z_+^{S_2^{C\,(q)}},$$

$$t_{r_2 c_o}^{(q)-} \in Z_+^{R_2^{C\,(q)}}, \quad t_{s_2 c_o}^{(q)+} \in Z_+^{S_2^{C\,(q)}}.$$

Nevertheless, if we omit the last constraint $\sum_{k=1}^{j} \lambda_k^{(q)} = 1$, we could handle the CRS assumption as well.

Notably, only controllable input and output values are incorporated into the objective function because those elements are within the control of the DMU's management. Any decrease in uncontrollable input values or increment in uncontrollable output values is rejected, as it is assumed that the management of the DMU has no control over these factors [48]. In addition, the slack of inputs and outputs in uncontrollable factors is also eliminated from the relevant constraint. This paper also considers the inputs and outputs are either fully controllable hybrid integer values or fully controllable hybrid integer values.

By resolving model (2), a DMU yields the optimal value with regard to the objective function. $\rho_{HIC}^{pSBM}$ . $x_{r_1 c_o}^{(q)}, y_{s_1 c_o}^{(q)}$, $x_{r_2 C_O}^{(q)} \in Z_+^{R_2^C\,(q)}$ and $y_{s_2 C_O}^{(q)} \in Z_+^{S_2^C\,(q)}$ demonstrate the set of controllable hybrid integer inputs and outputs. $t_{r_1 c_o}^{(q)-*}$, $t_{s_1 c_o}^{(q)+}$, $t_{r_2 c_o}^{(q)-*}$, $t_{s_2 c_o}^{(q)+}$ signify the slacks of controllable hybrid integer input excess and controllable hybrid integer output shortfall.

$\rho_{HIC}^{pSBM}$ illustrates a system's efficiency between zero (0) and one (1). If the resulted score is one, indicating the system is efficient. Alternatively, it is not efficient. Consequently, from the model (2), efficiency scores for the internal processes can also be obtained. Therefore, the efficiency score of process *k* is:

$$\rho_{HIC}^{(q)} = min. \frac{1 + \frac{1}{h}\sum_{q=1}^{p} \sum_{r^C=h^{(q-1)}+1}^{h^{(q)}} \frac{t_{r c_o}^{(q)-}}{x_{ro}^{(q)}}}{1 - \frac{1}{m}\sum_{q=1}^{p} \sum_{s^C=m^{(q-1)}+1}^{m^{(q)}} \frac{t_{s c_o}^{(q)+}}{y_{so}^{(q)}}}. \quad (3)$$

Based on models (2) and (3), the subsequent outcome can be precisely accomplished:

The system is efficient if and only if it is assessed as efficient in all processes.

Definition 3. A DMU is efficient if $\rho_{HIC}^{pSBM*} = 1$, and this condition is equivalent to all or some values with regard to the controllable hybrid integer input excess and/or output shortfall $t_{r_1 c_o}^{(q)-*}$, $t_{s_1 c_o}^{(q)+}$, $t_{r_2 c_o}^{(q)-*}$ and $t_{s_2 c_o}^{(q)+}$ are zero.

After developing this proposed model, the models can be solved to obtain the optimal solution by transforming their fractional programs into linear programs. The model mentioned above assumes variable returns-to-scale (VRS) for production, meaning the production frontiers are defined by the convex hull of the existing DMUs. However, the choice between CRS or VRS depends on the scale size of production. Given the significant variations in the production scale sizes of the DMUs in this study, we adopt the VRS assumption for our model.

## 4. Numerical Example

### 4.1. Effectiveness Assessment Using the Proposed DEA Model: A Case Study of a Malaysian Public University

The DEA model is a widely recognized method for assessing the relative efficiency of DMU. Beyond merely assessing efficiency, DEA also offers a robust framework for measuring effectiveness [49]. In the context of higher education, a public university can be conceptualized as a system that transforms input factors into performance results, such as graduate employability, research citations, and publications.

The administration of a university faculties operates with a parallel structure because universities manage multiple processes simultaneously. In this context, each faculty is considered a distinct DMU and is divided into two processes: Teaching (Process 1) and Research (Process 2), as depicted in figure 2. Each faculty allocates resources differently given to these processes. Prior to developing the model, it is necessary to establish a parallel structure that reflects the actual scenario, involving these two processes. From this structure, the study identifies the input and outcomes variables for inclusion in the parallel model. This study leverages the proposed DEA model to assess the effectiveness of a Malaysian public university comprising 26 faculties for the year 2020.

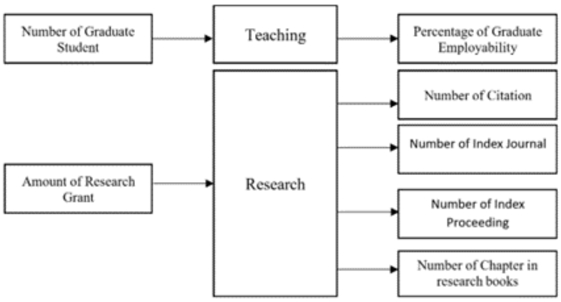


**Figure 2.** A public university with two parallel processes

Two factors were selected as inputs for the study. Firstly, the number of graduate students, classified as a controllable integer input [50]. Secondly, the amount of research grants, categorized as an uncontrollable non-integer input [51] due to its reliance on external sources such as government agencies, national and international foundations, and industry partners [52], beyond the direct control of the university's management. In contrast, five factors were identified as outputs. These include the percentage of graduate employability, classified as an uncontrollable non-integer output [35], and the number of citations, considered an uncontrollable integer output [53]. The number of citations is regarded as uncontrollable since it is determined by researchers' independent decisions on which works to cite based on the relevance and merit of the work rather than the influence of university management [54]. Additionally, three controllable integer outputs were selected: the number of indexed journals [55], the number of indexed proceedings [56], and the number of chapters in research books [57]. Table 1 provides a detailed categorization of the inputs and outputs for each process. These requirements and categorization should be considered in our approach as it offer a more accurate and informative assessment of performance within a network system.

**Table 1.** Categorization of the inputs and outputs

| **Process** | | | **Categorization of Factor/ Data** |
|---|---|---|---|
| 1 | Input | Number of graduate students | Controllable integer |
| | Output | Percentage of graduate employability | Uncontrollable non-integer |
| 2 | Input | Amount of research grants | Uncontrollable non-integer input |
| | Output | Number of citations | Uncontrollable integer |
| | | Number of index journals | Controllable integer |
| | | Number of index proceedings | Controllable integer |
| | | Number of chapters in the research book | Controllable integer |

The effectiveness scores for 26 faculties obtained from our approach model are presented in table 2 (the score is calculated using formula (1). Note that system effectiveness is denoted by ($p_0$), and processes effectiveness (teaching and research processes), denoted as ($p_1$) and ($p_2$), respectively. A faculty is considered effective when all processes within the parallel system are effective. The result shows that the system effectiveness score ranges from 0.085 to 1.000, indicating variations in overall effectiveness across the faculties. Several faculties (i.e., faculty 9, faculty 10, faculty 13, faculty 16) achieve maximum effectiveness across all processes. Where the rest of the faculties exhibit variations in effectiveness across processes, highlighting areas for targeted improvement with faculty 12 is the lowest. It can be seen that most of the teaching processes are not effective, and most research processes are effective for most of the faculties. In this context, the effectiveness score is heavily influenced by the performance of the teaching processes, implying that these faculties have room for improvement in teaching processes.

**Table 2.** Effectiveness score for 26 faculties

| Faculties | System Effectiveness, $\boldsymbol{p_0}$ | Teaching Process effectiveness, $\boldsymbol{p_1}$ | Research Processes Effectiveness, $\boldsymbol{p_2}$ | Aggregated Effectiveness |
|---|---|---|---|---|
| 1 | 0.751 | 0.751 | 1.000 | 1.000 |
| 2 | 0.302 | 0.302 | 1.000 | 1.000 |

| | | | | |
|---|---|---|---|---|
| 3 | 0.647 | 0.757 | 0.797 | 1.000 |
| 4 | 0.407 | 0.407 | 1.000 | 1.000 |
| 5 | 0.720 | 0.720 | 1.000 | 1.000 |
| 6 | 0.222 | 0.222 | 1.000 | 1.000 |
| 7 | 0.761 | 0.761 | 1.000 | 1.000 |
| 8 | 0.481 | 0.481 | 1.000 | 1.000 |
| 9 | 1.000 | 1.000 | 1.000 | 1.000 |
| 10 | 1.000 | 1.000 | 1.000 | 1.000 |
| 11 | 0.730 | 0.730 | 1.000 | 1.000 |
| 12 | 0.085 | 0.085 | 1.000 | 1.000 |
| 13 | 1.000 | 1.000 | 1.000 | 1.000 |
| 14 | 0.687 | 0.687 | 1.000 | 1.000 |
| 15 | 0.161 | 0.161 | 1.000 | 1.000 |
| 16 | 1.000 | 1.000 | 1.000 | 1.000 |
| 17 | 0.204 | 0.204 | 1.000 | 1.000 |
| 18 | 0.384 | 0.384 | 1.000 | 1.000 |
| 19 | 0.094 | 0.094 | 1.000 | 1.000 |
| 20 | 0.162 | 0.914 | 1.000 | 1.000 |
| 21 | 0.095 | 0.095 | 1.000 | 1.000 |
| 22 | 0.358 | 0.358 | 1.000 | 1.000 |
| 23 | 0.197 | 0.197 | 1.000 | 1.000 |
| 24 | 0.089 | 0.089 | 1.000 | 1.000 |
| 25 | 0.225 | 0.225 | 1.000 | 1.000 |
| 26 | 0.735 | 0.735 | 1.000 | 1.000 |

## 4.2. Comparisons with single SBM (Aggregated structure)

In order to clarify the importance of proposing the hybrid integer-valued and uncontrollable factors and into our alternative approach over the previous existing approach, a comparison is made with a single SBM [11] where the two processes (teaching and research processes in figure 1) are aggregated into a single "black box," as depicted in figure 3 in which the internal structure of the faculties is neglected. In this model, inputs are the sum of those in processes of teaching and research. Output is total which is measured as the sum of all outputs. We neglect the internal structure of the faculty. Table 2 lists the system effectiveness and aggregated effectiveness obtained by both parallel network SBM model and single SBM model respectively. There are some DMUs which are judged as effective in single SBM but ineffective in network parallel SBM. Therefore, our approach offers a more accurate and informative assessment of performance within a network system since it can give the details on each processes effectiveness.

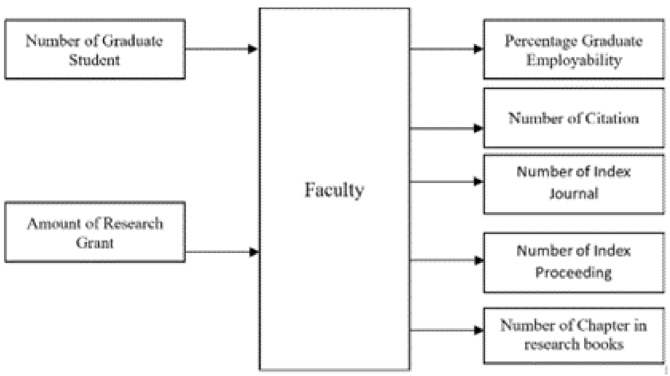


**Figure 3.** Aggregated structure.

Figure 4 further compares single SBM (aggregated effectiveness) with that of our alternative approach (system effectiveness) in graphical representation. To ensure that the proposed model has more discriminatory power, the effectiveness scores resulting from the model should be less than or equal to that obtained by applying the existing single SBM model. In our research, the effectiveness scores computed by the proposed approach are lower than or equal to those obtained from the previous single SBM approach. Therefore, it is confirmed that the proposed approach possesses more discriminatory power as compared to the single SBM approach.

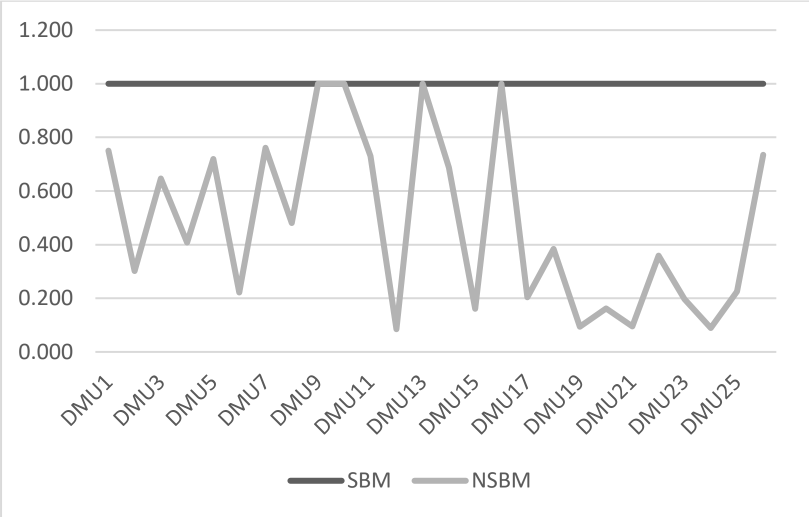


**Figure 4.** Comparison of single SBM [11] and our alternative approach.

## 5. Conclusion

This research developed a parallel SBM DEA model that concurrently addresses the hybrid integer-valued data and the uncontrollable factors. This alternative approach proves effective in accurately assessing real managerial scenarios where certain inputs and outputs are integer values and not regulated by the management of the DMUs. Since the model is based on an alternative approach of a parallel SBM, it may assess system-level and process-level effectiveness. As a numerical example, we utilized a dataset comprising 26 faculties in a public university. To summarize, this research introduces two key contributions. First, the model can distinguish between effective and ineffective DMUs by considering the operations of internal processes in a parallel network system. Second, the model addresses the managerial scenario where hybrid integer-valued data and uncontrollable factors are concurrently incorporated into the model. As systems grow more complex, the development and extension of this model have proven its relevance in practical applications such as banking, finance, transportation, telecommunications, government, and other sectors. This highlights the model's adaptability across industries beyond the specific case of a Malaysian public university. Finally, adding the concept of weighted approach in inputs and outputs by its importance in the proposed model would be an interesting subsequent research.

## 6. Declarations

### 6.1. Author Contributions

Conceptualization: S.N.Z.H., M.K.M.N., and R.K.; Methodology: M.K.M.N. and R.K.; Software: S.N.Z.H.; Validation: S.N.Z.H., M.K.M.N., and R.K.; Formal Analysis: S.N.Z.H., M.K.M.N., and R.K.; Investigation: S.N.Z.H.; Resources: M.K.M.N. and R.K.; Data Curation: M.K.M.N. and R.K.; Writing Original Draft Preparation: S.N.Z.H., M.K.M.N., and R.K.; Writing Review and Editing: M.K.M.N., R.K., and S.N.Z.H.; Visualization: S.N.Z.H.; All authors have read and agreed to the published version of the manuscript.

### 6.2. Data Availability Statement

The data presented in this study are available on request from the corresponding author.

## 6.3. Funding

The authors received no financial support for the research, authorship, and/or publication of this article.

## 6.4. Institutional Review Board Statement

Not applicable.

## 6.5. Informed Consent Statement

Not applicable.

## 6.6. Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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