Computer Simulation Control of High-Order Nonlinear Systems using Feedback

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Abstract

The relevance of the topic stated in this research is the need to develop and implement methods of computer modelling of these control systems. The purpose of this research is the search for opportunities for controlling nonlinear systems and the creation of a computer model for controlling nonlinear systems. The basis of the methodological approach in this research work is a combination of methods of theoretical and applied research of general principles of construction of computer models of control of high order nonlinear systems by means of feedback. In the course of the research work, the results were obtained, indicating the effectiveness of the development of an algorithm for finding the control of tracking a given reference signal of nonlinear systems and nonlinear systems with time delay. An algorithm has been developed to find a control that can effectively track the output signal of a nonlinear system behind a given reference signal. In addition, a scientific analysis of the tracking and stabilization errors of nonlinear systems and time-delayed nonlinear systems has been carried out depending on the control parameters, and graphical representations of a computer model of numerical experiments performed according to the control algorithms have been presented. It is established that the output control problem for a nonlinear system is to obtain a feedback control that forces the controlled output signal of the nonlinear system to asymptotically track the reference signal. The practical significance of the obtained results lies in the possibility of their use in the creation of computer models of process control with feedback.

Keywords: Output tracking, Systems with delay, State controller, Lyapunov-Krasovsky functional, Algorithm for finding control, Tracking error analysis

1. Introduction

It is well known that the problem issues of global output tracking control of nonlinear systems are quite complex and involve a number of important aspects in the field of nonlinear control. Over the last few decades, the issues of output tracking control for nonlinear systems have been thoroughly investigated. At the same time, the results obtained do not allow fully studying the effect of time delay. Time delay as a system phenomenon is observed everywhere in many practical models, particularly in mechanical, chemical, biological and electrical systems [1], [2]. The presence of a time delay can cause significant performance degradation or cause the entire system to fail [3]. Practical solution of the problem, put in the subject of this scientific research, can be of significant importance for the activity of industrial production facilities of the Republic of Kazakhstan, which use automatic control systems (ACS) of activity, the principle of operation of which is based on the control of nonlinear systems with feedback. For today, a qualitatively new stage of the decision of problem aspects of synthesis of ATS is development and introduction of computer models of management of such systems, in the form of optimum decision of problems of synthesis of an algorithm of management for the given object. Initially, they were considered and defined exclusively as functions of time, which

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was essential for implementation in the form of programme control systems, after that – as functions of output control variables representing a closed system with a set of feedbacks.

The output control problem for a nonlinear system is to obtain a feedback control that forces the controlled output of the nonlinear system to asymptotically track the reference signal. Also, the objectives of this research study include: developing an algorithm for finding the control of tracking a given reference signal of nonlinear systems and nonlinear systems with time delay; using the developed algorithm to determine the control that will track the output signal of the nonlinear system behind a given reference signal; depending on the parameters of the control, conducting tracking and stabilization error analysis of nonlinear systems and nonlinear systems with time delay, creating a computer model of the numerical experiments of the nonlinear systems and nonlinear systems with time delay, creating a computer model of the numerical experiments of the nonlinear systems and nonlinear systems with time delay [4].

At the same time, Alimhan et al. considered in their work the general principles of construction of a system of global control of production tracking in high-order nonlinear systems with time-varying delays [5]. The scientists note that the problems of global input tracking control of nonlinear systems have a high degree of complexity. According to the authors, further scientific research in this direction will contribute to expanding the range of scientific ideas regarding the prospects of application of the class of nonlinear systems with time-varying delays under weaker conditions for nonlinear systems. Alimhan et al. reviewed a number of problematic aspects of output tracking for a class of nonlinear, uncertain systems with high-order time delay [6]. According to the scientists, control design should be considered one of the most relevant topics in nonlinear systems' theory today. In this case, the main problem is the possibility of constructing a feedback control law capable of providing tracking capability with a maximum, controllable reference signal output. The current study attempts to go further into the particular impacts of time delays on the performance of nonlinear systems by building on these fundamental ideas. In contrast to earlier research, which focused on building feedback control laws without offering a unified approach that could be used to nonlinear control design, this study suggests a new algorithm for tracking control that takes temporal delays into account. By offering a thorough analysis and a solution for output tracking and stabilisation in nonlinear systems impacted by time delays, this work aims to close the gap left by previous research.

For their part, a group of scientists consisting of Mei et al. conducted a scientific study of the general regularities of exponential stabilization by means of feedback control with high-order time delay [7]. According to the researchers, there are a significant number of systems subject to random delays and time-delay effects in various fields of modern science and engineering. At the same time, in many practical stochastic models the delay coefficients of these systems do not meet the conditions of linear growth, which necessitates a detailed study of the general principles of control of high-order nonlinear systems by feedback. Their results required a thorough investigation of the principles of control for high-order nonlinear systems under feedback. This work extends that investigation by creating computer models that replicate numerical tests for these kinds of systems. By attempting to provide a quantitative and visual depiction of system behaviours under various management scenarios, these models aim to make managing the intricacies of time-delayed systems more approachable.

A group of research scientists consisting of Jiang et al. conducted a scientific study of a number of problematic aspects of stabilization of control of distribution processes of systems with time delays and intermittent feedback [8]. The authors note that the decision probability distributions tend to converge within a stationary distribution. According to the scientists, the solution probability distributions of a stochastically discontinuous controllable high-order nonlinear system tend to converge to a stationary distribution.

The aim of this research study is to investigate the real perspectives of controlling nonlinear systems in the context of creating a computer model for controlling nonlinear systems. In order to achieve the goal, the following objectives were set: development of an algorithm for finding the tracking control of a given reference signal of nonlinear systems and nonlinear systems with time delay; development of detailed computer models to simulate numerical experiments for nonlinear systems and nonlinear systems with time delays to provide a visual and quantitative representation of the system's behavior under different control scenarios; investigation of the impact of time delays on the performance of nonlinear systems, particularly in the context of output tracking and stabilisation to develop strategies to mitigate their adverse effects; exploration of practical applications of the developed control algorithms for industrial production facilities in the Republic of Kazakhstan. This includes evaluating the potential benefits of implementing these algorithms in automatic control systems used in various industrial processes.

2. Materials and Methods

The methodological basis of this research work was the combination of a set of methods of theoretical and applied research of general principles of construction of computer models of control of high order nonlinear systems by means of feedback, taking into account the delay on the time parameter. The theoretical basis of this research work was the analysis of the results of scientific research of a number of Kazakh and world authors, aimed at studying a wide range of problem aspects of computer modelling of technological processes of control of high-order nonlinear systems with feedback. Besides, in the mentioned theoretical base of this research work the results of scientific research in the field of estimation of possibilities and prospects of practical application of these models in various spheres, for the solution of purely practical problems presented in various fields of modern science and technology were included (figure 1).

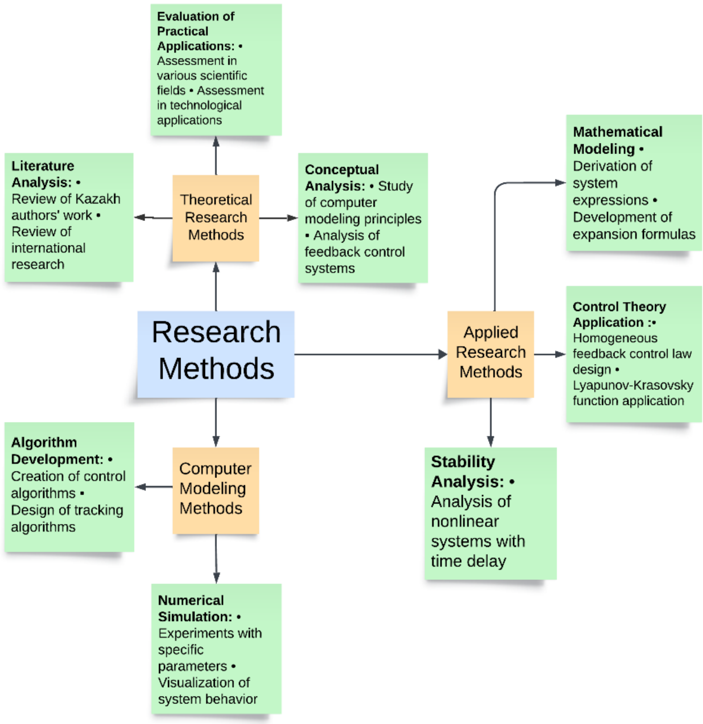

Figure 1. Visualization of the methodology

The application of a combination of the above-mentioned methods of scientific research allowed characterizing the problem of finding and tracking the output of high-order nonlinear systems. An expression depicting the general form of a high-order nonlinear system with delay was derived, which was represented in expression (1). Then, through the definition of one-way function expressions, expressions for the expansion $\Delta s(x)$ were obtained, the calculation of which is performed through formula (2). Also, the expression for the definition of the function (3), homogeneous, represented in degree n, was obtained.

The use of the chosen combination of scientific research methods has allowed obtaining expression (11), showing the sequence of construction of the homogeneous feedback on the actual position of the controller for a nominal nonlinear system. In turn, this has provided theoretical data for the design of the stabilizer of the homogeneous feedback nonlinear system represented in expression (9). In control systems, a feedback control law that demonstrates homogeneity features in the feedback functions is referred to as homogeneous feedback. In particular, under a dilatation (scaling) transformation, a feedback control law is deemed homogenous if the control inputs and the system states meet specific scaling features. For its qualitative representation, the Lyapunov-Krasovsky function describing homogeneous, nonlinear systems with time lag was applied. The Lyapunov-Krasovsky function is an effective instrument for analysing the stability of time-delayed systems. Through the incorporation of the system's state history, it expands upon the conventional Lyapunov technique and enables a thorough stability analysis even when delays occur.

The application of a set of theoretical research methods allowed obtaining the determination of a number of parameters necessary to create an algorithm for finding the control of tracking a given reference signal of nonlinear systems and nonlinear systems with time delay. In addition, through the application of the mentioned method of scientific research, the theoretical assumptions necessary for the effective solution of the set problem were obtained. The combination of the noted assumptions with the applied analysis allowed determining the main regularities of the development of the control algorithm for tracking the input signal of a nonlinear system with feedback.

There are a number of drawbacks to the output tracking control approach that has been suggested for nonlinear systems with temporal delays. The model's dependence on theoretical presumptions may oversimplify actual situations, and nonlinear systems with temporal delays present a substantial difficulty. The findings' practical relevance and generalisability are questionable since the efficacy of suggested management measures may be impacted by changes in system characteristics, outside disruptions, and dynamic environmental circumstances. It's possible that the homogeneous feedback assumption leaves out important dynamics, which might impair the model's capacity to make accurate forecasts or implement useful control strategies in real-world settings. For real-time applications, the computational cost of putting the suggested algorithms into practice and carrying out numerical trials could be too much to handle, which would make it less practical for control or decision-making applications that need to respond quickly. Overcoming these obstacles is necessary for the model's practical implementation in industrial environments, such as those found in Kazakhstan's manufacturing facilities. To fully realise the potential advantages of the model and its relevance in improving control system performance in real-world settings, practical validation and adaptation of the theoretical findings are required.

Creation of the control algorithm of the input signal tracking process implied the study of the general principles of the design of the feedback tracking control of the output signal of the nonlinear system after a given reference signal. In order to effectively solve this stated problem, a transformation coordinate was introduced to qualitatively track the practical output in the study of the feedback problem for high-order nonlinear systems with feedback. Through the application of an applied computer model building method, models of computer simulation experiments were obtained using specific numerical parameters obtained according to the established control algorithms. This provided a visual representation of the nature of the change in the magnitudes of the estimated error with changes in the stabilization of time-delayed nonlinear systems.

3. Results

The global problem of searching for and practically tracking the output of nonlinear systems is one of the most significant and complex in the field of nonlinear control. By placing certain conditions on the growth of the system and the order of power, the practical problem of tracking the output data from the system (1) has been well studied, and significant practical results have been obtained. A high-order nonlinear system with delay can be represented as (1):

$$
\begin{aligned}
z_1(t) &= x_2(t)^{p1} + \phi_1(t, x(t), x(t-d), u(t)),\\
z_{n-1}(t) &= X_n(t)^{P_{n-1}} + \phi_{n-1}(t, x(t), x(t-d), u(t)),\\
Z_n(t) &= u + \phi_n(t, x(t), x(t-d), u(t)),\\
y(t) &= x_1(t),
\end{aligned}
\tag{1}
$$

where, t – time; p1 – power, positive odd integer; u(t) – control input; x(t–d) – the delayed state; d – delay; y(t) – the output of the nonlinear system at time t; x(t) – the current condition or configuration of the system in its n-dimensional state space; x1(t) – the output of the nonlinear system at time t; ϕ – the nonlinear disturbance that affects the state variables of the nonlinear system; y(t) – tracking error; tx(t)=(x1(t), … xn(t))T Є Rn, and Є R, and y(t) Є R describes the state of the system, with control of the input and output parameters, respectively.

The constant parameter d≥0 reflects the system delay time for i=1, … n, while the initial condition of the system is x(Ө)=ϕ0(Ө), Ө Є [0, d]. In this case, the conditions ϕi(.) represent nonlinear disturbances, which are presented as continuous, unknown functions and pi Є R≥1odd={p/q Є [0, ∞):p and q are odd integer values, while the parameters p≥q} (i=1, … n–1) indicate that the nonlinear system has a high order.

Equation (1) does not take into account the effect of time delay, which has a significant impact on the performance of nonlinear systems in the conditions of their practical application. This makes it highly important to develop issues of tracking, output and stabilization of high-order nonlinear systems with time delays. However, due to the lack of a unified research method that can be effectively applied to nonlinear control design, this problem has not been studied sufficiently, in addition, there are numerous significant problems that have not yet been solved [9], [10]. Currently, the use of the Lyapunov-Krasovsky method with a time delay has made it possible to significantly improve control theory and methods for practical solution of the problem of stabilizing the delay of a nonlinear system. In the case where the nonlinearities contain a time delay to exit the tracking problem, some interesting results have also been obtained. However, contributions are taken into account only in special cases, such as pi equals one or a constant delay for system (1), when the parameter pi has a larger value. When the system does not take into account time delays, the problem becomes more complex and remains unresolved [5].

For the set of the following coordinates x=(x1, … xn), Є Rn, as well as a number of variable values r=(r1, rn) of positive, real numbers, the following definitions are introduced. The expansion Δs(x) is a mapping, which is calculated using (2):

$$\Delta_s(x) = (S^{r_1}x_1, \dots s^m x_n), V(x) = (X_1, \dots x_n) \in R^n, V(x) > 0, \quad (2)$$

where, s – a scalar used in the dilatation mapping; r1,…,rn – the coordinate values.

For ease of notation, the dilatation parameter is defined as Δ=(r1, … rn). The function V Є C (Rn, R) is homogeneous, represented to the power n, if there is a real value of n Є R, in particular (3):

$$V\left(\Delta_{s^n}(x)\right) = S^n V\,(x_1, \dots x_n), V(x) \in R^n - \{0\}. \quad (3)$$

The vector field f Є C (Rn, Rn) is considered to be homogeneous, represented to the power n, if the components fi are homogeneous to the degree n+ri for all values of i corresponding to (4):

$$f_i\left(\Delta_{s^r}(x)\right) = s^{r+ri} f_i(x_1, \dots x_n), V(x) \in R^n, Vs > 0. \quad (4)$$

This expression is valid for values of I lying in the range I=1, … n. The homogeneous p-norm can be defined using (5):

$$(x)_{\Delta,p} = (\Sigma_{i=1}^{n}(x_i)^{p/ri})^{1/p}, V(x) \in R^n, p \geq 1. \quad (5)$$

For simplicity, the notation (x)=Δ is used instead of (x)Δ,2.

Assumption 1. For the existing constant values of C1 and C2, as well as for n≥0, the following (6) is valid:

$$\phi_i\left(t, x(t), \mu(t-d), u(t)\right) < C_{1(x_1(t)^{(ri+n)/ri} + x_i(t)^{(ri+n)/ri} + x_1(t-d_1)^{(ri+n)/ri})} + C_2, \quad (6)$$

where, ϕi – nonlinear disturbances; C1, C2 – positive constants that provide upper bounds for nonlinear disturbances ϕi; z(t–d1)=x(t–d1), x(t–d2), … x(t–dn), r1=1, ri+1pi=ri+n>0, i=1, … n and pn=1.

In many practical systems, nonlinear disturbances ϕi(t,x(t), μ(t−d), u(t)) arise due to environmental factors, sensor inaccuracies, or external influences. Assumption 1 asserts that these disturbances are bounded by a polynomial expression involving the current state xi(t), delayed states xi(t−di), and constants C1 and C2. These constants provide upper bounds for ϕi(t,x(t), μ(t−d), u(t)), reflecting practical limits beyond which the disturbances do not significantly affect system behaviour. These bounds are derived from empirical observations or theoretical models of the system, ensuring that the disturbances do not lead to unstable conditions or unpredictable responses. The conditions r1=1 and ri+1=pi=ri+n>0 specify the power and delay characteristics of the disturbances. They ensure that the influence of delayed states xi(t-di) and the nonlinearity of the disturbances are adequately captured in the model. This allows for a comprehensive analysis of how past states affect current system behaviour, critical in high-order nonlinear systems where delays and nonlinearities can amplify instability if not properly bounded. This assumption ensures that the system's dynamics remain within predictable bounds, crucial for stability analysis and control design.

Assumption 2. The reference signal yr(t) undergoes continuous differentiation. In addition, the constant D>0 is known, which determines the validity of (7):

$$\mathrm{y_r(t) + Y(t) \leq D, V(t) \in [0, \infty).} \tag{7}$$

where: yr(t) – reference signal.

Continuous differentiation of yr(t) ensures that the reference signal and its derivatives are well-defined and smooth over time. This property is fundamental in control systems as it allows for precise tracking and adjustment of the system's output to meet desired performance criteria. Without continuous differentiation, abrupt changes or discontinuities in yr(t) could lead to instability or overshooting in control responses. The inequality yr(t)+Y(t)≤D establishes a boundary on the sum of the reference signal yr(t) and any deviations Y(t) from this reference. The constant D>0 serves as a maximum allowable deviation, ensuring that the control system operates within safe limits relative to the reference signal. This constraint is crucial in practical applications to prevent excessive error accumulation or control saturation, thereby maintaining system stability. Assumption 2 guarantees that the control system can effectively manage deviations Y(t) from yr(t) without exceeding the predefined threshold D. This feasibility condition ensures that the control design remains effective and predictable under varying operational conditions, enhancing the system's reliability and performance over time. By imposing Assumption 2, engineers can develop control algorithms and strategies that are robust and resilient to disturbances or uncertainties in the system. The bounded nature of yr(t) and Y(t) allows for practical implementation of feedback control systems across different applications, from industrial processes to autonomous vehicles, where precise tracking of reference signals is critical for operational success.

Formula 7 must hold true within the defined interval V(t)∈[0,∞), ensuring that the control system can effectively manage deviations from the reference signal yr(t) without exceeding the predefined threshold D. To follow the practical conclusion when studying the feedback problem for high-order nonlinear systems with feedback, the transformation coordinate (8) is introduced:

$$z_1 = x_1 - y_r, z_i = \frac{x_1}{L^{Ki}}, i = 2, \dots n, v = \frac{u}{L^{Kn+1}}, \tag{8}$$

where, K1=0, Ki=(Ki–1+1)/Pi-1, i=2, … n and L≥1 parameters to clarify the scale, which will be applied later.

This will allow expression (1) to be presented in the following form (9):

$$\begin{gathered} \dot{z}_i = Lz_{i+1}^{Pi} + \phi_1(t, z(t), z(t - d_i), v), i = 1, \dots n - 1, \\ \dot{z}_n = Lv + \phi_n(t, z(t), z(t - d_n), v), \\ y = z_1, \end{gathered} \tag{9}$$

where, ϕ1(t,z(t), z(t–d1), v)=¥1(t,z(t), z(t–d1), v)–yr, ϕi(t,z(t), z(t–di), v)=¥i(t,z(t), z(t–di), v)/Lx, i=2, … n.

Further, using assumption 1, and also taking into account the fact that L≥1, (6) can be presented in the following form (10):

$$\begin{gathered} \phi_1(t_1, z(t), z(t - d_1) \leq C_{1(z_1(t)^{(r1+n)r1} + z_1(t-d_1)^{(r1+n)/r1})} + C_2, \\ \phi_i(t, z(t), z(t - d_i), v) \leq C_1 L^{1-vi} \Sigma_{j-1(z_j(t)^{(ri+n)/ri} + z_j(t-d_j)^{(rj+n)/rJ})}^{i} + C_2/L^{ki}, i = 2, \dots n, \end{gathered} \tag{10}$$

where, vi=min{1–kj(ri+n)/rj+ki, 2≤j≤i, 1≤i≤n}>0; C1>0 and C2>0 constants.

It follows that in the future a homogeneous approach will be used to construct global feedback in a high-order nonlinear system. Firstly, homogeneous feedback is built on the actual state of the controller for a nominal nonlinear system, nonlinearity is not taken into account (11):

$$\mathrm{\phi_i\ (.), i = 1, \dots n - 1, \dot{z}_i = Lz\ {}_{i+1}^{Pi}, i = 1, \dots\ n - 1, \dot{z}_n = Lv, y = z_1.} \tag{11}$$

where: ϕi(.) – nonlinear disturbances.

Using an approach similar to that, a homogeneous feedback stabilizer can be designed for expression (9). Its description is presented in the following theorem [11].

Theorem. For a certain, real value n≥0, a homogeneous feedback controller is specified, represented in the power of n, such that the nonlinear system (11) is asymptotically stable. For the proof, an inductive argument is used to most fully reflect the homogeneous stabilizer of the nonlinear system (11). Let ϴ1=z1α/r1–z1*α/r1, where z1*=0, and α≥max{1, n+ri} be positive values. Next, the Lyapunov function (12) is selected:

$$V_1 = W_1 = \int_{*z1}^{z1} (s^{n/r1} - z_1^{*n/r1})^{(2n-r-ri)?n} ds. \quad (12)$$

From (11) it follows (13):

$$V_1^* \leq nL\Theta_1^2 + L\Theta_1^{(2n-r-ri)/n}(z_2^{p1} - z_2^{*p1}). \quad (13)$$

If assuming that the value of the interval is k–1, a certain positive Lyapunov function Vk-1 and the complex of virtual controllers is defined as follows (14):

$$z_1^* = 0, \Theta_1 = z_1^{n/r1} - z_1^{*n/r1}, z_i^* = -Ƀ_{i-1}^{r1/n}\Theta_{i-1}^{r1/n}, \Theta_i = z_i^{n/ri} - z_i^{*n/ri}. \quad (14)$$

For values of Ƀi>0, 1≤I≤k–1, (15) is obtained:

$$V_{k-1}^* \leq -(n-k+2)L\Sigma\Theta_1^2 + L\Theta_{k-1}^{\frac{2n-r-r1}{n}}\left(z_k^{pk-1} - z_k^{*pk-1}\right). \quad (15)$$

Expression (15) is also valid for the step value k, since there is a positive Lyapunov function, presented in the form of the following (16):

$$V_k(\text{Ꮇ}_k) = V_{k-1}(\text{Ꮇ}_k - 1) + W_k(\text{Ꮇ}_k). \quad (16)$$

When expressing the virtual controller z*k-1=–Ƀkrk+1/nϴrk+1/n the following definition (17) is valid:

$$V_k^* \leq -(n-k+1)L\Sigma_{i-1}^k\Theta_1^2 + L\Theta_k^{(2n-r-rk)/n}(z_{k+1}^{pk} - z_{k+1}^{*pk}). \quad (17)$$

Taking into account the above (17), it is possible to conclude that at the nth step there is a feedback controller, which is expressed in the following form (18):

$$V = -Ƀ_n^{rn+1/n}\Theta_n^{rn+1/n} = -(\Sigma_{i=1}^n Ƀ_i z_i^{n/ri})^{rn+1/n}. \quad (18)$$

Taking into account the value of C1, as well as the positive definite Lyapunov function, the following definition is obtained (19):

$$V_n^* \leq -L\Sigma_{j=1}^n \Theta_j^2, \quad (19)$$

where, ϴ1=zin/ri–zi*n/ri, and Ƀi*=Ƀn, … Ƀi, i=1, … n – positive constants.

Thus, the closed nonlinear system presented in (18) is asymptotically stable. Consider a high-order nonlinear system (20):

$$x_1^* = x_2^3 + x_2^3/3(1 + x_2^2), x_2^* = x_3 + {}^1\!/_4\,(x_2^{1/3}\sin x_1^2 + x_2^3)$$
$$x_3^* = u + \frac{1}{7x_3}, y = x_1 - y_r, \quad (20)$$

where, p1=3; p2=2; p3=1; yr(t)=(sin(t))3 – the desired reference signal.

It should be noted that yr(t) does not imply the presence of restrictions in ensuring effective tracking of the output signal of a nonlinear system behind a given reference signal. In the proposed computer model, the following values of the initial states of the system are established: x1(0)=–7, x2(0)=5, x3(0)=10, x*2(0)=–10, x*3(0)=50. With a clearly defined gain parameter M=450, the stable value of the resulting error is within 0.2. This implies that after the system has settled and reached a steady state, the difference between the system's output x1(t) and the reference yr(t) is consistently 0.2 or less. The calculation of steady-state error typically involves allowing the system to stabilise and then measuring the difference between the reference and actual outputs. In a situation where there is an increment to a value of M=4500, there is a consistent decrease in the magnitude of the tracking error to a value within 0.1. This suggests that increasing the gain improves the system's ability to track the reference signal more accurately. Higher gain values often lead to faster response and smaller steady-state errors in control systems. The steady-state tracking

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error of a high-order nonlinear system is consistently within 0.2. The magnitude of the tracking error $y(t)=x1(t)-yr(t)$ with the parameter M=4500 is consistently within 0.1.

The fact that the steady-state error remains within 0.2 demonstrates that the system, with a moderate gain M=450, can effectively track the reference signal under normal operating conditions. This is crucial for applications where precise control over the output is required. Increasing the gain to 4500 and observing a reduced error of 0.1 highlights the sensitivity of the system's performance to the control parameter M. Higher gains can enhance tracking accuracy but may also introduce risks such as instability or oscillations if not carefully tuned. The system performs well in controlled situations, but real-world applications may provide difficulties that need to be resolved. This is indicated by the statement of practical constraints as well as problems with robustness and dependability. These might be parameter errors, outside disruptions, or nonlinearities that the model does not completely account for.

The suggested model provides a sophisticated method of controlling high-order nonlinear systems with time delays, but its practical application is restricted by issues with robustness and reliability in real-world applications, mathematical complexity, difficulties estimating parameters, and assumptions about system behaviours. The model's effective implementation in industrial control systems will depend on how these issues are resolved. With a focus on both theoretical and practical elements, table 1 described the organized method used in the study to design and verify an algorithm for managing high-order nonlinear systems with temporal delays.

Table 1. The development of the algorithm for controlling high-order nonlinear systems with time delays

No.	The sequence of actions	Description
1	Problem Definition	Define the high-order nonlinear system with time delays and establish the practical problem of output tracking.
2	System Representation	Represent the high-order nonlinear system with delay using formula 1, which include system dynamics and disturbances.
3	Expansion and Dilatation Mapping	Define the expansion Δs(x) and dilatation parameter Δ=(r1, … rn) to transform system coordinates.
4	Homogeneous Function Definition	Introduce the homogeneous function V represented to the power n, and the homogeneous vector field f.
5	Assumptions	State Assumptions 1 and 2 regarding nonlinear disturbances and reference signal differentiation.
6	Coordinate Transformation	Introduce transformation coordinates z1, zi, and v to qualitatively track the practical output.
7	System Reformulation	Reformulate the system in terms of transformed coordinates, considering the nonlinear disturbances.
8	Homogeneous Approach	Apply the homogeneous approach to construct global feedback in the high-order nonlinear system.
9	Feedback Control Law	Develop a feedback control law that stabilizes the nonlinear system using a homogeneous feedback stabilizer.
10	Lyapunov-Krasovsky Function	Use the Lyapunov-Krasovsky function for stability analysis of the time-delayed system.
11	Numerical Simulations	Conduct numerical simulations to validate the control algorithms and demonstrate tracking performance.

An efficient solution to the ultimate output control problem for a nonlinear system is to evaluate the feasibility of obtaining feedback control that directs the controlled output of the high-order feedback nonlinear system to asymptotically track a reference signal. The practical application of the Lyapunov-Krasovsky functional is necessary to form and run an algorithm for adjusting the scaling factor for a general nonlinear closed-loop system. Precise determination of this coefficient makes it possible to qualitatively provide global tracking of an entire class of uncertain, nonlinear systems over a specific time interval. This has significant practical significance for the subsequent application of computer models for controlling high-order nonlinear systems using feedback in solving a wide range of practical problems.

The simulations of the high-order nonlinear system with time delays, as represented by (20), yielded several noteworthy outcomes. Initially, the system was defined with specific parameters p1=3, p2=2, and p3=1, indicating varying degrees of nonlinearity across its components. The reference signal yr(t)=(sin(t))3 imposed a continuous differentiation requirement, essential for smooth tracking and stability in control systems (Assumption 2). This ensured that the system's response to yr(t) remained manageable, preventing excessive deviations that could compromise stability.

Throughout the simulations, the system's initial states were set as x1(0)=–7, x2(0)=5, and x3(0)=10, with corresponding derivative initial conditions x*2(0)=–10 and x*3(0)=50. These initial conditions were chosen to reflect typical starting points in practical applications, influencing how the system responded to control inputs and disturbances.

An essential aspect of the simulations was the evaluation of the tracking error y(t)=x1(t)–yr(t) under varying gain parameters M. It was observed that as M increased from 450 to 4500, the steady-state tracking error decreased significantly, illustrating the sensitivity of the system's performance to the control gain. This sensitivity highlighted the trade-off between system responsiveness and stability, where higher gains improved tracking accuracy but also introduced potential for oscillatory behavior or instability if not carefully managed.

Comparing these outcomes with theoretical expectations, particularly those derived from assumptions such as Assumption 1, which bounds nonlinear disturbances ϕi(t, x(t), x(t–d), u(t)), provided valuable insights. Assumption 1 ensured that these disturbances remained within predefined limits C1 and C2, preventing them from causing unpredictable behaviour or instability. The theoretical framework also incorporated the use of Lyapunov-Krasovsky methods to analyse stability, confirming that the system could achieve asymptotic stability under appropriate feedback control strategies.

However, the practical implementation of the model faced challenges typical of complex nonlinear systems with time delays. Issues such as robustness in real-world applications, the mathematical complexity of parameter estimation, and the reliance on idealized assumptions about system behaviour (e.g., perfect differentiation of yr(t)) posed significant hurdles. These challenges underscored the need for further refinement in control algorithms and adaptation to specific industrial contexts where reliability and robustness are paramount.

In conclusion, while the simulations demonstrated promising results in terms of tracking performance and stability, translating these findings into practical control strategies for high-order nonlinear systems with time delays requires addressing remaining theoretical and practical challenges. Future research should focus on enhancing robustness, refining parameter estimation techniques, and validating the model's performance across diverse operational conditions to ensure its applicability in real-world scenarios.

Significant practical implications emerge from the study of regulating high-order nonlinear systems with time delays for Kazakhstani industrial production facilities, particularly those that depend on ACS. The potential for improved industrial control system performance and stability is one of the main advantages of this research. For instance, reaction rates in a chemical plant may be quite nonlinear, and instability or oscillations may arise from delays in the feedback control loop. The plant may obtain more accurate control over the reaction processes, lowering the risk of instability and enhancing product consistency, by implementing the suggested model. The controllability of nonlinear systems with delays is an essential skill in the energy sector, especially in the areas of power distribution and generation. Even with changing load circumstances, the model in a power plant can optimise the regulation of generators and transformers to provide stable voltage and frequency levels throughout the grid. Delays in the control system can cause bottlenecks and inefficiencies in automated warehouse systems, where conveyor belts, automated guided vehicles (AGVs), and sorting systems must cooperate. These systems may more efficiently synchronise their activities, cutting down on wait times and improving warehouse efficiency overall, by using the suggested control measures. Furthermore, Kazakhstan's mining sector may benefit most from the suggested plan. Large machinery and intricate procedures that are prone to temporal delays and nonlinear behaviours are used in mining operations. For instance, to

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optimise productivity and safety, drilling and blasting activities need to be precisely timed and coordinated. The precision of these processes may be increased by the sophisticated control algorithms created in this study, which would result in more effective resource extraction and lower operating expenses.

Table 2 highlights the possible influence of the computer model on industrial applications, technical breakthroughs, and control theory academic research. It also illustrates how these implications extend to different practical, theoretical, and educational realms.

Table 2. The computer model's application in managing nonlinear systems with and without time delays

Industrial Applications	Theoretical Contributions	Real-world Implementation	Technological Advancements	Educational and Research Impact
Enhances control system efficiency in industrial production settings	Provides theoretical frameworks and algorithms for nonlinear system control	Supports adaptation to real-world industrial environments, particularly in Kazakhstan	Advances in computational modelling for handling high-order nonlinear dynamics	Contributes to educational curricula in control theory and nonlinear dynamics
Improves stability and tracking of reference signals in nonlinear systems	Validates control strategies through numerical simulations and theoretical proofs	Offers insights into practical challenges and considerations in applying control algorithms	Integrates feedback control laws with homogeneous properties for scaling transformations	Facilitates further research in nonlinear control systems with time delays

In conclusion, this research has enormous practical implications for Kazakhstani industrial production facilities. The suggested model can improve the stability, effectiveness, and dependability of several industrial processes by tackling the difficulties associated with regulating high-order nonlinear systems with temporal delays. There are several applications for this study, ranging from automated manufacturing and chemical plants to mining and power generation. These applications have the potential to significantly reduce costs and enhance operating efficiency.

4. Discussion

The issues of building computer models of control of high-order nonlinear systems in the context of sliding mode control on the basis of composite nonlinear feedback were considered in the scientific study by Mondal and Mahanta [12]. Scientists pay attention to the fact that the use of digital computers to create control models of nonlinear systems with time delays implies the transmission of the control signal at fixed time intervals at a certain delay. The authors conclude that the application of a nonlinear control law ensures the constancy of the time delay and tracking of the output signal of the nonlinear system behind a given reference signal [11], [13]. The scientists' conclusions are in line with the results of this scientific study, since the constancy of time delay in controlling complex, high-order nonlinear systems is a mandatory factor for the effective functioning of the mathematical model.

At the same time, Zhang et al., D. Li and T. Li considered a number of problematic aspects of adaptive, asymptotic event-based tracking for stochastic nonlinear delayed systems [14], [15]. In their research study, the scientists concluded that fuzzy logic is highly effective in estimating unknown time delays, as the development of computer control models for high-order nonlinear systems with feedback requires taking into account time delays in the presence of significant sensor and actuator faults. The scientists' conclusions are consistent with the results that were obtained in this research work, as they reflect the need to take into account the time lag when assessing the performance of the theoretically developed computer model in real, practical conditions.

In turn, Song et al. considered a number of problematic aspects of synchronizing semi-Markovian spiking neural networks with sensor nonlinearity using feedback systems [16]. According to the researchers, the models of the system undergo changes in the presence of time delays and when they are varied. As a result of the scientific study, the scientists concluded that computer modelling of the control of high-order nonlinear systems with feedback implies the presence of time delays caused by external disturbances, in particular, in the transmission of information or constraints on the action of physical components [17], [18]. The researchers' conclusions are in line with the results that were obtained in this research work, as they confirm the findings regarding the effect of time delays on the performance of high-order nonlinear systems with feedback.

Zirkoni, Yu and Li aimed at studying the general principles of building a reliable adaptive feedback control of uncertain fractional-order systems with time delay on the input, draws attention to the fact that the creation of a computer model of control of high order nonlinear systems by building feedback implies taking into account the ability of the regulator to cope with the approximator error and unknown disturbance [19], [20]. According to the scientist, among the key advantages of the control approach he proposes in his research paper is that a fractional order system with input delay is effectively handled by introducing a new variable as a fractional order system without delay. The scientist's opinion is fully supported by the results that have been obtained in the course of this research work in view of the fact that it displays the advantages of computer modelling of the control of high-order nonlinear systems with feedback, taking into account the influence of external perturbations on the nature of the functioning of the system as a whole [21], [22].

Zhao et al. considered in a scientific paper a wide range of problematic aspects of practical application of an intelligent coordinated feedback controller, the principle of operation of which is based on the use of computer models of data processing [23]. The researchers express the opinion that when building this kind of computer models, a hierarchical online control mechanism with feedback, which includes a set of regulators, should be used. In this case, the functionality of the computer model is provided by the possibility of efficient processing of numerous requests per unit of time. The researchers' conclusions are consistent with the results obtained in this research paper, as they emphasize the relationship between the efficiency of computer model functioning and its ability to process a significant amount of information in a given time.

Li et al., Van and Zhou addressed the issues of optimization of computer modelling of control processes of high order nonlinear systems, for high performance and large-scale computing systems [24], [25]. According to the group of authors, the increase in the total number of resources in high performance and large-scale systems contributes to the complexity of the computer models used to control high-order nonlinear systems with feedback. At the same time, nonlinear design methods are not suitable for solving global output tracking problems. The scientists' opinion seems to be controversial and needs practical verification, since not always the increase of resource volumes entails the complication of control models of high-order nonlinear systems.

For their part, Geronimo et al. considered in their scientific work a number of problematic aspects of building computer models of control of high order nonlinear systems for the purpose of modelling the processes of functioning of urban transport networks [26]. Scientists came to the conclusion that the significant growth of the population in large cities, along with an increase in the number of vehicles, has caused the problem of ensuring the safety of urban transport. Theoretically and practically, problems of this kind can be successfully solved by applying methods of mathematical modelling of these processes, using high-order nonlinear systems with feedback. The researchers' conclusions are confirmed by the results of [27]. Obtained in this scientific work, in the context of the analysis of the prospects of solving the set problems with the help of methods of mathematical, computer modelling.

At the same time, Ruth and Cleveland considered the general principles of dynamic computer modelling of the processes of oil reserves depletion, through the application of control models of high order nonlinear systems with the help of feedback [28]. In the course of their scientific research, the scientists came to the conclusion that in order to build a theoretical computer model describing the above processes, it is necessary to take into account a combination of a wide range of factors describing natural, physical, natural phenomena in the form of specific mathematical dependencies. This will make it possible to express the basic regularities in the form of high-order nonlinear systems with feedback and specific time frames. The opinion expressed by scientists and related to the theoretical side of the issues addressed in this research paper is consistent with the results obtained in it, which represent their practical representation [29], [30], [31].

Turan et al., Namazov et al., investigated the principles of applying computer modelling techniques for nonlinear feedback control systems in order to optimize material consumption [1], [32]. In the course of their research, the scientists concluded that when creating a simulation model, real-time data from the production line should be used. In this case, any time delays affect the efficiency of the computer model of control of nonlinear systems with feedback. The opinion of the researchers is supported by the results obtained in this research paper, due to the fact that it reflects the relationship of the effect of time delays on the performance of the computer model of control of nonlinear systems in general [33].

Thus, the discussion of the results that were obtained during the performance of this research in the context of their analytical comparison with the results and conclusions of other researchers who have carried out the development of a wide range of problem aspects of building computer models of control of high-order nonlinear systems with the use of feedback has demonstrated their fundamental correspondence on a number of investigated issues.

Future studies should focus on improving robust control methods, researching multi-input multi-output control techniques, resolving real-world implementation issues, improving system modelling, and fusing data-driven and advanced sensing technologies. The progress made will not only broaden the scope of current research but also facilitate the efficient implementation of high-order nonlinear systems in many real-world scenarios, guaranteeing enhanced efficiency, dependability, and flexibility.

5. Conclusion

As a result of this scientific research, it was possible to establish that the global, practical problem of tracking a given reference signal of nonlinear systems with a time delay can be effectively solved theoretically and practically. The creation of a computer model that describes the control processes of high-order nonlinear systems using feedback opens up significant prospects for control theory in general, and is also of great importance in the context of the prospects for controlling individual objects in particular. The resulting algorithm for finding the control for tracking a given reference signal of nonlinear systems and nonlinear systems with a time delay can be very effective in practical development in various areas of modern science and technology. A practical solution to the problem of signal regulation for a nonlinear system at the output assumes the possibility of obtaining control in the presence of feedback, which forces, by controlling the signal of the nonlinear system at the output, to regulate the process of asymptotic tracking of the reference signal.

The problem of performing global tracking of a class of uncertain time delays of a high-order nonlinear system can be effectively solved by choosing an appropriate order of tracking actions. The use of the Lyapunov-Krasovsky functional contributes to the creation of an algorithm for tuning the scaling factor for a general closed-loop system. The correct choice of this coefficient allows for global tracking of an entire class of uncertain, nonlinear systems over a specified time period. This has significant practical significance for the subsequent practical application of computer models for controlling high-order nonlinear systems using feedback.

The prospects for further scientific research in the direction defined within the scope of this work are determined by the possibility of practical application of the results obtained in order to ensure the effective functioning of industrial and other equipment, as well as a number of other devices from various fields of modern science and technology, the control of which is based lies in the application of high-order nonlinear systems with feedback.

6. Declaration

6.1. Author Contributions

Conceptualization: G.B., N.T., A.T., A.D., A.A., and Y.U.; Methodology: A.T., N.T., and G.B.; Software: A.A.; Validation: N.T., A.T., and G.B.; Formal Analysis: A.T., N.T., and G.B.; Investigation: G.B.; Resources: A.D.; Data Curation: A.D.; Writing Original Draft Preparation: G.B., N.T., A.T., A.D., A.A., and Y.U.; Writing Review and Editing: A.T., G.B., and N.T.; Visualization: A.A.; All authors have read and agreed to the published version of the manuscript.

6.2. Data Availability Statement

The data presented in this study are available on request from the corresponding author.

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The authors received no financial support for the research, authorship, and/or publication of this article.

6.4. Institutional Review Board Statement

Not applicable.

6.5. Informed Consent Statement

Not applicable.

6.6. Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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