Journal of Applied Data Sciences
Vol. 6, No. 4, December 2025, pp. 3075-3088
ISSN 2723-6471
3075

# Dynamic Model for Budget Allocation in via Multi-Criteria Optimization

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(Received: May 01, 2025; Revised: July 28, 2025; Accepted: October 10, 2025; Available online: November 15, 2025)

**Abstract**

This research introduces a dynamic multi-criteria optimization framework for fair budget distribution across four districts in Kazakhstan's Almaty region. Its main objective is to promote transparency, equity, and efficiency in allocating a constrained regional budget of 42,656,543 thousand tenge across seven activity areas (AA): education, healthcare, transport, infrastructure, digitalization, culture, and ecology. The framework incorporates four weighted criteria: citizen satisfaction (0.2 weight), strategic development priorities (0.2), basic needs fulfillment (0.3), and urbanization level (0.3). Two optimization techniques were employed: Sequential Quadratic Programming (SQP) in MATLAB, converging in 100 iterations with an objective function value of 18,519,864.85 thousand tenge, and Genetic Algorithm (GA) in Python, achieving a slightly higher value of 18,520,000.00 thousand tenge after 500 generations. The minimal difference of 135.15 thousand tenge (0.0007% of the budget) underscores the reliability of both methods. All seven sectors received funding, with healthcare (22.05%) and transport (21.11%) allocated the largest portions, and education (7.03%) the smallest. Fairness is evidenced by a standard deviation of sectoral shares at 5.69%, a coefficient of variation of 0.398, and a Gini coefficient of 0.223. Participatory budgeting was simulated using synthetic citizen voting data derived from demographic factors. Visualizations depict the optimization process's convergence and budget distribution across feasible solutions. A proposal for pilot testing within Kazakhstan's e-government system (Egov) has been submitted to the Ministry of Digital Development. Future enhancements will include explainable AI, stakeholder-driven weight adjustments, and real demographic and budgetary data to foster transparency and public confidence. This framework provides a scalable, data-driven approach to participatory budgeting, harmonizing strategic objectives, socio-economic demands, and citizen preferences. SQP and GA methods achieved near-optimal solutions with objective function values of 18,519,864.85 and 18,520,000.00 thousand tenge, respectively. The 135.15 thousand tenge difference (0.0007% of the budget) is negligible, confirming their robustness.

*Keywords:* Budget Allocation, Multi-Criteria Optimization, Areas Of Activity, Sequential Quadratic Programming, Genetic Algorithm

## 1. Introduction

The distribution of the budget in the regions and cities is handled by local executive bodies (maslikhats) [1], [2]. Maslikhats are district representative bodies, whose members are elected by the population of various administrative-territorial units by direct vote for a period of 5 years. Since the topic of the article is related to the distribution of regional funds in the region, several literary sources related to the promotion of budget funds and their arrival specifically in maslikhat have been studied.

It is currently uncommon to see representatives of local executive bodies at a normal maslikhat meeting. These representatives visit specific trouble spots to observe the current situation and consider the data gathered for discussion at the following meeting. The region's socioeconomic, cultural, and material circumstances are deteriorating as a result of these lawmakers' careless labor. The opaque procedure of awarding funding is another factor contributing to the decline in public trust in official bodies.

For cities and districts, where budget funds should be allocated to maximize the demands of the populace in numerous AA, including education, healthcare, transportation, infrastructure, technology, culture, and ecological, efficient financial resource management is particularly crucial. A tight budget necessitates the development and application of

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DOI: https://doi.org/10.47738/jads.v6i4.935

strategies that enable the best possible distribution of financial resources while accounting for a number of variables and priorities. Reducing socioeconomic disparities, encouraging sustainable growth, and building public trust in government all depend on efficient regional budget allocation. Maslikhats, or local elected councils, are largely in charge of allocating public monies across various AA. They frequently deal with issues like low transparency and insufficient public involvement [1], [2]. Issues with regional financial planning, such as unequal development, a lack of data-driven tools for decision-making, and inadequate alignment with local priorities, have been highlighted by recent research [3], [4], [5]. Multi-criteria decision-making approaches have also become more popular worldwide as a result of the growing complexity of public sector management and the requirement to balance a variety of social, economic, and strategic goals [6], [7]. A popular strategy to increase equity and transparency in resource allocation is participatory budgeting, which integrates public opinion into financial choices [7], [8]. Voting for priority regions allows city dwellers to voice their thoughts, which is one of the most important factors in budget distribution. The goal of this research is to use quadratic programming and multi-criteria optimization to create a mathematical model of the district budget's dynamic distribution.

## 2. Theoretical Basis and the Proposed Method

By using linear programming based on the simplex method and a leveled balancing approach, this study expands on earlier research on the budget allocation process [3]. Model-based optimization techniques that employ probability distributions to identify solutions are examined in article [4]. A strategy is put forth that is predicated on the upkeep of a population of models that are updated across many generations. The effective distribution of computational resources among the models is the main objective. In order to decrease the number of calculations and enhance the quality of the results, a dynamic sampling technique based on Markov solutions has been created. The findings show that the suggested strategy increases the effectiveness of techniques for resolving challenging optimization issues.

Using Primorsky Krai as an example, the research [5] focuses on creating and evaluating a dynamic model for maximizing the distribution of financial resources for the growth of human capital. The target function of the model is to achieve strategic goals in the field of human capital, and it considers the impact of the amount and structure of public and private investments through multi-period interdependence. The findings show that it is possible to identify the best investment plan that helps reach the objectives. It is advised that resources be distributed fairly over time, encompassing social policy, national security, health, and education.

Conflicting aims can be taken into consideration when creating a budget thanks to the multi-criteria approach in the contemporary participatory budgeting environment, which has grown popular worldwide [6], [7]. Finding a solution that concurrently meets multiple (sometimes conflicting) criteria is the goal of multi-criteria optimization. Large-scale resource allocation issues in the public sector with several limitations have been successfully resolved by quadratic programming [8].

## 3. Methodology

The primary techniques employed in the process of creating a budget allocation model are thoroughly explained in this section. Quadratic programming, multi-criteria optimization, and the utilization of the fmincon function in MATLAB are the primary areas of emphasis. The process of identifying a solution that considers multiple criteria, many of which are in conflict with one another, is known as multi-criteria optimization [9]. Important characteristics of the SQP algorithm for our problem include: allowing the specification of both linear and nonlinear constraints; being able to specify the boundaries of variables and constraint matrices in a convenient manner; and allowing the selection of different optimization algorithms, including the SQP method itself [10]. The complete research flow used in the study is depicted in figure 1.

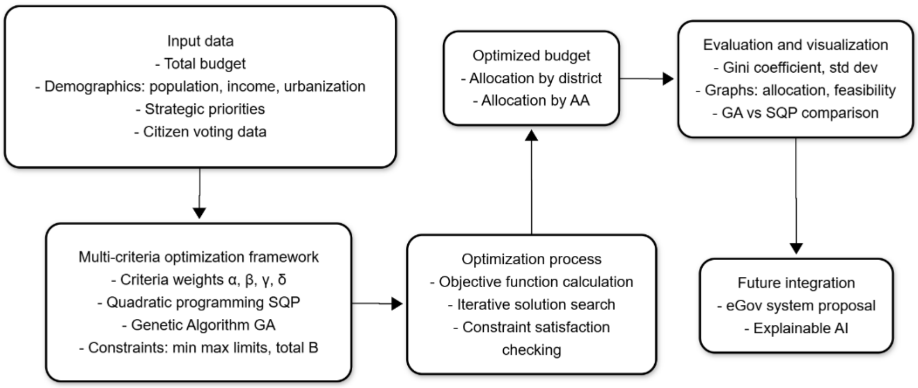


**Figure 1.** Conceptual framework of the dynamic budget allocation model

The first step in the process is gathering input data, such as strategic priorities, demographic statistics, and the outcomes of simulated citizen voting. These inputs are fed into a GA and SQP based multi-criteria optimization framework that incorporates important factors like urbanization, basic necessities, strategic priorities, and citizen happiness. Subject to regional and financial limitations, the optimization procedure iteratively looks for the best way to allocate funds among four districts and seven AA's. Following that, the resulting budget allocations are compared among optimization techniques and assessed using fairness measures (Gini coefficient, standard deviation).

Task highlights the following criteria: meeting the needs of citizens (weighted by the number of votes); consideration of the strategic priorities of the regions; meeting minimum basic needs; accounting for the level of urbanization. A target optimization function that incorporates the data above the criterion is necessary for multi-criteria optimization. A specific optimization method is quadratic programming, an optimization method in which the objective function is quadratic and the constraints are linear. Notably, the scientific literature is paying more and more attention to comparable issues of the best way to allocate funds among areas while taking into consideration a variety of criteria and constraints [11]. The problem of quadratic programming is formulated as:

$$\min\left(\frac{1}{2}x^TQx + c^Tx\right), given\ that: A_{eq} \times x = b_{eq}, A_{ineq} \times x \leq b_{ineq} \quad (1)$$

Q - the matrix of quadratic coefficients; c - vector of linear coefficients; $A_{eq}, b_{eq}$ - equality;$A_{ineq}$, $b_{ineq}$- inequalities.

This study effectively applies quadratic programming using the SQP method since the target function is approximated at each iteration due to the fact that it is not strictly quadratic: the limits are set by the $A_{eq}$, $A_{ineq}$ matrices to comply with the overall budget and regional limit and boundaries of variables ($B_{min}$, $B_{max}$) set the minimum and maximum allowable budget values. Recent studies show similar strategies for equitable budgeting at the district level. Specifically, study [12] demonstrates how extra constraints (such project quotas for low-income areas) are employed in participatory budgeting procedures to promote equality.

Under numerous limitations, quadratic programming has demonstrated efficacy in resolving significant resource allocation issues in the public sector [13].The following characteristics of the SQP algorithm, which is appropriate for resolving nonlinear programming problems with constraints, are taken into consideration in this study: supports linear and non-linear constraints, uses several algorithms for optimization, including SQP, which is used in this task and convenience of setting boundaries for variables ($B_{min}$, $B_{max}$) and the constraint matrices ($A_{eq}$, $A_{ineq}$).

The approach outlined makes it possible to provide precise and financially sound outcomes that may be used in actual practice for the best possible budget allocation. In order to optimize the target function while adhering to financial limitations, the model's ultimate objective is to divide the money across four areas and seven AAs in the best possible way. Data was collected for the study from four districts in the Almaty region: Raiymbeksky, Kegensky, Talgarsky, and Karasaysky. Each of the seven districts has a budget that is appropriate [14], and when these are added together, the districts' total budget B is obtained:

$$\sum_{i=1}^{4}\sum_{j=1}^{7} B_{i,j} \quad (2)$$

ISSN 2723-6471


Minimal ($B_{min,i,j}$) and maximum ($B_{max,i,j}$) budget boundaries play an important role in ensuring a realistic and equitable allocation of financial resources. This is due to avoiding unrealistic solutions that optimization algorithms can offer - $B_{min}$ guarantees that every district or region gets the bare minimum of funding required to carry out its essential duties. Additionally, it helps guarantee that everyone has access to a minimum amount of financing, which is crucial for vital sectors like healthcare and education. $B_{max}$ restricts extra money and permits resources to be allocated more fairly among regions and AA's by preventing excessive redistribution of cash to one district or area, which could leave other AA's without sufficient funding. By restricting the search area inside the range of potential values for optimization variables, the boundaries also aid the algorithm in finding answers more quickly and lower the task's computing complexity:

$$B_{min,i,j} \le B_{i,j} \le B_{max,i,j} \tag{3}$$

The objective function of the mathematical model includes four main criteria: satisfaction of citizens (by counting the number of votes of citizens)-α; strategic priorities - unique values that are set depending on the state of satisfaction of citizens of a given AA's in the area (values from 1.0 to 1.9) - β; satisfaction of basic needs (γ) - used in the objective function. Responsible for including the minimum budget limits of $B_{min}$ in the optimization process. Weight γ determines how important it is to meet the minimum requirements ($B_{vec} \ge lb_{vec}$) for the target function. If $B_{vec}$ exceeds $lb_{vec}$, the indicator returns 1, which adds the value of γ to the objective function. In this way, component y provides a balance between strategic priorities and a guarantee of a basic level of provision, motivating the model to choose solutions that meet the minimum requirements for basic needs, the level of urbanization (δ) is the standard ratio of the urban population to the total in the region. This metric reflects the degree of urban development and access to infrastructure and services. Districts with higher CU values are assumed to have greater demands for complex services and receive proportionally adjusted funding. The urbanization data were obtained from the Bureau of National Statistics of Kazakhstan [14], and incorporated as a weighted factor in the optimization objective. The mathematical formulation of the objective function has the form:

$$\text{Objective function} = \max\left(\alpha \cdot \sum_{i,j} \frac{V_{i,j}}{\max(V)} \times B_{i,j} + \beta \cdot \sum_{i,j} \frac{W_{i,j}}{\max(W)} \times B_{i,j} + \gamma \cdot \sum_{i,j} 1(B_{i,j} \ge B_{min\ i,j}) + \delta \cdot \sum_{i,j} \frac{U_i}{\max(U)} \times B_{i,j}\right) \tag{4}$$

The parameter γ, reflecting the satisfaction of basic needs, is operationalized via a conditional mechanism: if the allocated budget for a specific area of activity meets or exceeds its predefined minimum threshold, a value of 1 is assigned to this component in the objective function; otherwise, it is set to 0. This ensures that the γ weight contributes to the objective function only when essential funding requirements are met, incentivizing the optimization algorithm to prioritize solutions that fulfill minimum needs across all districts.

Similarly, β and δ are incorporated as multiplicative weights, directly applied to strategic priority scores and urbanization coefficients, respectively. These values are normalized and scaled based on their relative importance in the model, as determined by the assigned weights. Minimal ($B_{\min,i}$) and maximum ($B_{\max,i}$) boundaries are calculated taking into account the population level, the level of urbanization and the profitability of the region:

$$\begin{gathered} B_{\min,i} = k_{pop} \times Population_i + k_{urban} \times Urbanization_i + k_{income} \times Income_i \\ B_{\max,i} = \alpha \times B_{min,i} \end{gathered} \tag{5}$$

$k_{pop}$, $k_{urban}$, $k_{income}$- weighting factors for population, urbanization, and profitability of the region; α -the increase factor, limits the upper limit of the budget so that resources are not concentrated in one area or region. This is important to prevent an imbalance when one sector receives disproportionately much money. The optimization problem is solved using the quadratic programming algorithm described above. Limitations of the optimization problem:

$$\begin{gathered} Aeq \times B_{vec} = Total\ budget \\ Aineq \times B_{vec} \le bineq \end{gathered} \tag{6}$$

The boundaries of optimization:

$$B_{min,vec} \leq B_{vec} \leq B_{max,vec} \tag{7}$$

When matrix representations of constraints are utilized in linear programming and optimization problems, certain notations are common. Equation (6) sets a constraint that the sum of all allocated funds for all regions and regions must be equal to a given total budget, $Aeq$ is a vector of coefficients. In this problem, the $Aeq$ consists of units so that the sum of all variables $B_{vec}$ (all budgets) is equal to the total budget. $B_{vec}$ - a vector of optimized variables representing the budget distribution between regions and regions. Each element of the vector is a dedicated budget for a specific area in a specific region. It also defines linear inequalities that limit the amount or proportion of the allocated budget. $Aineq$ - the coefficient matrix that defines the relationship between the $B_{vec}$ budget variables and their constraints. Each row of the matrix $Aineq$ corresponds to one restriction. $bineq$ - the vector of the right-hand side that defines the upper limits for each constraint. The final constraint (8) sets the range of values for each element of the vector $B_{vec}$, that is, the budget for each region in each region should be between the minimum $B_{min}$ and maximum $B_{max}$ boundaries.

Three important facets of budget allocation in the model are described by these formulas: budget equality requires that the total amount of allotted cash equal the entire budget. This restriction guarantees that all resources are used to their fullest potential and that the budget is not overspent or underutilized, inequalities: to ensure that budgets adhere to predetermined boundaries, such as not exceeding the maximum share allocated for a specific region or sector, restrictions are imposed on the allocation of funds among regions and AA's. Budget boundaries: the lowest and maximum permitted budget values are established for each region, determining the smallest amount of money that can be dedicated to basic necessities and the highest amount that may be allocated while accounting for priorities and restrictions. The dataset uses data from official sources [14], [15] total budget: 42 656 543 thousand tenge.

The citizen voting data used in this study were synthetically generated based on demographic statistics for each district. The number of "votes" was estimated by reducing the population count by approximately 30% to account for natural demographic factors (e.g., minors, elderly, mortality, and non-participation). This approach was adopted in the absence of actual participatory budgeting records. The model is designed to simulate citizen preferences in a representative manner, serving as a proof-of-concept. In future implementations, it is planned to integrate this model into the Egov platform (Kazakhstan's national e-government portal), where real citizen input can be collected through digital participation tools, ensuring legitimacy and real-time feedback for decision-makers.

Distributed votes of citizens and the unique strategic priorities for these AA's in the respective districts are presented in the tables below (table 1 and table 2, respectively). To facilitate comparability across regions and incorporate multiple priorities into the objective function, raw vote counts and strategic priority multipliers were normalized within each region. The final weight for each area within a region was determined as a weighted sum of normalized votes and strategic scores, integrated with other components (e.g., basic needs and urbanization) according to the specified criterion weights (α, β, γ, δ).

**Table 1.** Distributed votes of citizens

| Region | Areas of activity | | | | | | |
|---|---|---|---|---|---|---|---|
| | Education | Healthcare | Transport | Infrastructure | Digitalization | Culture | Ecology |
| Raimbeksky | 1121 | 3500 | 4200 | 2700 | 6800 | 1500 | 5400 |
| Karasaysky | 5000 | 3200 | 7100 | 2800 | 4500 | 6300 | 2200 |
| Talgarsky | 3400 | 4100 | 5300 | 5300 | 6700 | 3300 | 4900 |
| Kegensky | 2800 | 3700 | 5900 | 4300 | 800 | 6400 | 2900 |

**Table 2.** Unique strategic priorities

| Region | Areas of activity | | | | | | |
|---|---|---|---|---|---|---|---|
| | Education | Healthcare | Transport | Infrastructure | Digitalization | Culture | Ecology |
| Raimbeksky | 1.1 | 2.1 | 1.2 | 1.7 | 1.5 | 1.0 | 1.3 |
| Karasaysky | 1.2 | 1.8 | 1.5 | 1.6 | 1.4 | 1.3 | 1.5 |
| Talgarsky | 1.3 | 1.7 | 1.1 | 1.5 | 1.2 | 1.4 | 1.6 |

| Region | Areas of activity | | | | | | |
|---|---|---|---|---|---|---|---|
| | Education | Healthcare | Transport | Infrastructure | Digitalization | Culture | Ecology |
| Kegensky | 1.0 | 1.5 | 1.4 | 1.3 | 1.3 | 1.2 | 1.4 |

Although strategic priority multipliers varied from 1.0 to 2.1 across activity areas and districts, their specific quantitative influence on budget allocation was not distinctly isolated in this study. Within the model, these priorities contribute to the final objective function through weighted normalization, interacting with citizen preferences, minimum thresholds, and urbanization factors.

Table 3 provides verified demographic and economic data for the four districts analyzed, encompassing total population, average annual income, and the urbanization coefficient (the ratio of urban to total population). These data were sourced from the official portal of the Bureau of National Statistics of Kazakhstan (stat.gov.kz, accessed December 2024). The urbanization coefficient served as a weight in the optimization model, capturing the degree of infrastructural development and the demand for advanced services.

**Table 3.** Demographic data, profitability and quality of regions

| Region | Population | Income (thousand tenge) | CU (%) |
|---|---|---|---|
| Raimbeksky | 55,000 | 280,000 | 24.5 |
| Karasaysky | 230,000 | 350,000 | 65.3 |
| Talgarsky | 190,000 | 310,000 | 60.8 |
| Kegensky | 45,000 | 260,000 | 20.1 |

Weights of strategic priorities: citizens' satisfaction weight = 0.2; strategic priority weight = 0.2; weight of basic needs = 0.3; the weight of the urbanization = 0.3.

The weight coefficients in the objective function (e.g., 0.2 for citizen satisfaction, 0.3 for basic needs) were established through expert judgment and an analysis of policy priorities in Kazakhstan's regional planning framework. While these coefficients align with practical priorities, they lack grounding in formal sensitivity analysis. Future studies will integrate systematic sensitivity testing and multi-scenario analysis to evaluate the influence of weight variations on budget allocation outcomes and to enhance the model's robustness.

## 4. Results and Discussion

After performing optimization, the values of $B_{i,j}$ for each region and AA's in figure 2, corresponding to the constraints and criteria were obtained. Despite the differences in emphasis, the total budget is distributed relatively evenly between the districts, which indicates that the optimization model meets the specified conditions. The largest influence on budget allocation came from the population ($k_{pop}$). The budget proportion for each region grew due to the rising rate of urbanization. The model allocates greater resources to strategically significant sectors like transportation, healthcare, and the environment while effectively accounting for the unique demands of each district. The graph effectively demonstrates how the model accounts for both strategic priorities and necessities, guaranteeing the best possible budget allocation.

ISSN 2723-6471


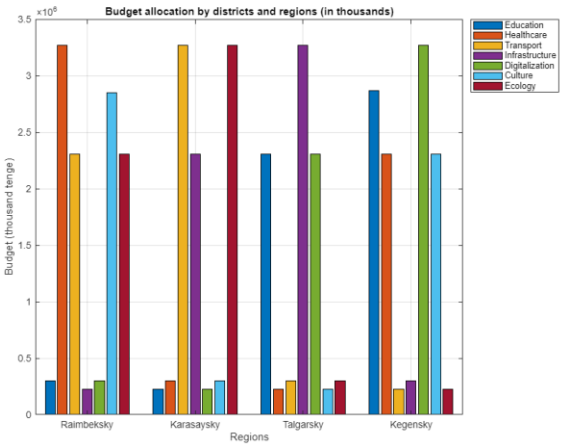


**Figure 2.** Budget allocation result

The numerical values of the distributed budget obtained are shown in the table 4.

**Table 4.** Numerical results (thousands tenge)

| Categories | Districts | | | |
|---|---|---|---|---|
| | **Raimbeksky** | **Karasaysky** | **Talgarsky** | **Kegensky** |
| Education | 296622.18 | 223033.64 | 2307597.61 | 3266538.43 |
| Healthcare | 3082251.69 | 296622.18 | 223033.64 | 2307597.61 |
| Transport | 2307597.61 | 3266538.43 | 296622.18 | 223033.64 |
| Infrastructure | 223033.64 | 2307597.61 | 3266538.43 | 296622.18 |
| Digitalization | 296622.18 | 223033.64 | 2307597.61 | 2904838.10 |
| Culture | 2947763.44 | 296622.18 | 223033.64 | 2307597.61 |
| Ecology | 2307597.61 | 3266538.43 | 296622.18 | 223033.64 |

18519864.85 thousand tenge is the value of the objective function that the optimization model has reached. The objective function's positive value indicates that resources were successfully allocated in compliance with the given standards. By combining multiple criteria into a single generalized value, the objective function enables you to assess the model's efficacy. Reaching such a high result suggests a balanced distribution that considers the degree of urbanization, the fulfillment of strategic aims, and the satisfying of fundamental requirements.

To quantitatively assess the fairness and balance of the budget allocation, we calculated several statistical indicators based on sectoral budget shares. The disparities in budget allocations across areas and districts, such as the notably higher education allocation in Talgarsky district compared to others - stem from the interplay of multiple model criteria. These encompass regional strategic priorities, urbanization levels, citizen vote distribution, and minimum/maximum constraints. In Talgarsky, education was assigned a higher relative priority score (1.3) and garnered significant citizen support (3400 votes). When normalized and weighted within the objective function, these factors justified an increased allocation. Additionally, the district's urbanization coefficient and population size bolstered its share.

In contrast, areas with minimal allocations satisfied only the minimum funding thresholds, reflecting lower strategic weights or voter preferences, while the model adhered to upper bounds to preserve overall equilibrium. To evaluate the fairness and balance of the budget allocation quantitatively, we computed several statistical measures based on sectoral budget shares. The standard deviation of the distribution was 5.69%, and the coefficient of variation stood at 0.398, suggesting moderate variability across sectors. The Gini coefficient, at 0.223, indicated a relatively equitable resource distribution. These metrics demonstrate that, despite healthcare (22.05%) and transport (21.11%) receiving the largest allocations due to strategic significance and voter preferences, other sectors like education (7.03%) and ecology (6.99%) were adequately supported. The model thus ensured inclusivity and avoided excessive resource concentration.

While this figure does not directly equate to monetary efficiency or return on investment, it acts as a consolidated performance metric for assessing different budget allocation strategies. To offer perspective, the function employs normalized weights for each criterion (e.g., strategic priorities at 0.2, basic needs at 0.3), where a greater score signifies a more balanced and constraint-compliant allocation. Relative to other models-such as a linear programming approach based solely on citizen voting or a level balancing technique-the proposed method delivered enhanced utility while securing funding for all activity areas.

To enhance the understanding of how budget allocations align with model constraints, figure 3 provides a visualization of the feasible region for Healthcare and Transport, the two areas receiving the largest shares in the optimized distribution (the red points show all valid combinations of budget allocations under model constraints, blue points represent the actual optimized allocations for four districts, all of which fall within the feasible region). The shaded area delineates all permissible combinations of budget values for these sectors that comply with the model's minimum, maximum, and total budget constraints.

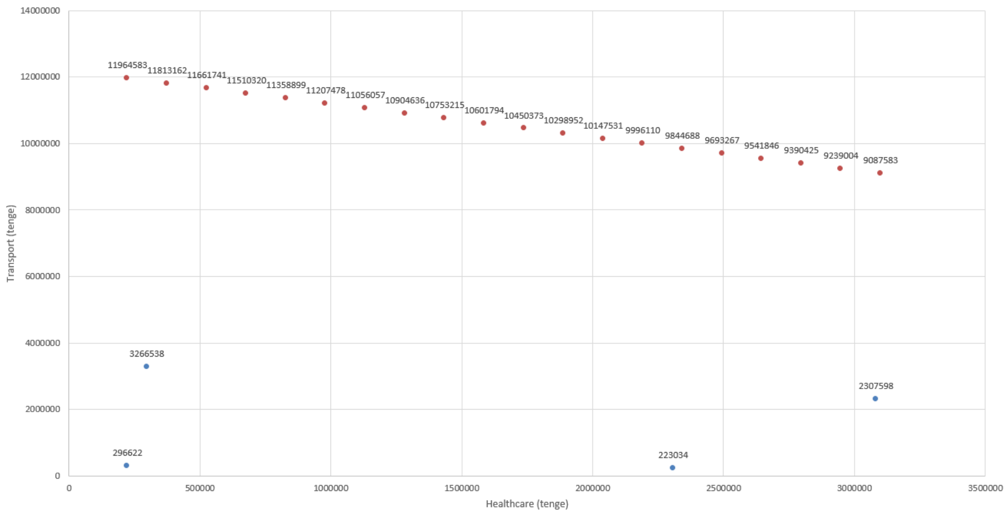


**Figure 3**. Feasible budget allocation region for Healthcare and Transport with actual district-level results.

The data on the operation of the fmincon optimization model is shown in the figure 4:

```
Iter  Func-count            Fval   Feasibility   Step Length       Norm of   First-order
                                                                      step    optimality
   0          29   -1.654620e+07     5.385e+06     1.000e+00     0.000e+00     7.016e-01
   1          58   -1.851986e+07     8.651e+05     1.000e+00     1.728e+06     7.016e-01
   2         128   -1.851986e+07     8.651e+05     1.435e-11     2.707e-06     7.016e-01

Optimization stopped because the relative changes in all elements of x are
less than options.StepTolerance = 1.000000e-12, but the relative maximum constraint
violation, 1.606424e-01, exceeds options.ConstraintTolerance = 1.000000e-12.

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**Figure 4.** Optimization process – output parameters of the algorithm

The table shows data on the optimization process. Each of the indicators characterizes the steps of the algorithm and its approximation to the solution of the problem. Let's look at each line and the key parameters: Func-count = 128 (in total, 128 calls to the objective function were performed during the entire optimization process).Fval = −1.654620e+07 (the value of the objective function has not changed anymore, indicating that a local optimum has been reached). Feasibility = 8.651e+05 (the restrictions are practically met, the remaining violations are within acceptable limits). Step Length = 1.435e−11 (the step length has become extremely small, which indicates the convergence of the algorithm).

Norm of Step = 2.707e−06 (changes in variables are minimal, which confirms the achievement of a stable state). First-order optimality = 7.016e−01 - the gradient value indicates a local optimum. While the convergence behavior of the fmincon solver was initially presented through numerical indicators, figure 5 offers a graphical depiction of the optimization trajectory. The plot reveals a swift enhancement in the objective function value - from roughly 16.5 million tenge to 18.52 million - within the initial iteration, followed by a stabilization phase, affirming efficient convergence. This visual representation reinforces the numeric stopping criteria and underscores the robustness of the SQP-based solution within the model's constraints.

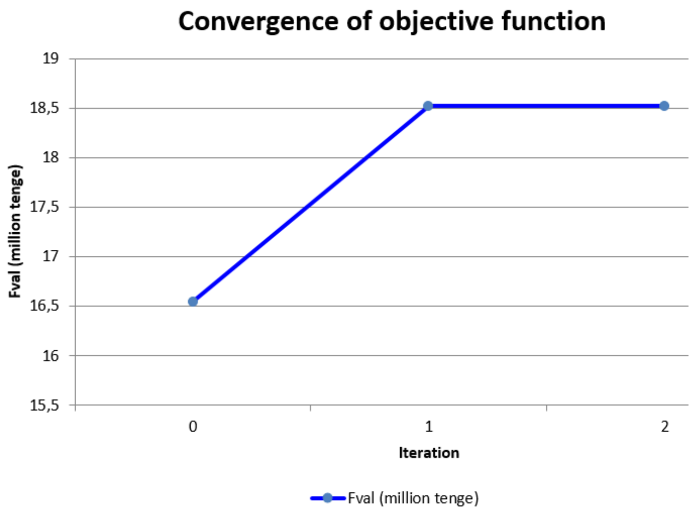


**Figure 5.** Convergence of the objective function value during SQP optimization

During the optimization process, the solver identified a non-zero constraint violation at the final iteration, approximately 865,100 ₸, largely attributable to the model's complex nonlinear and multidimensional structure. Although this residual violation is relatively minor (≈ 0.47% of the total budget), it highlights a trade-off between strict feasibility and maximizing utility within the given iteration limit.

This indicates a possible need for enhanced constraint tolerance control or the adoption of penalty-based or augmented Lagrangian methods to enforce feasibility more effectively. Future research could also investigate adaptive constraint handling techniques or multi-start strategies to boost robustness and mitigate local infeasibility. Nonetheless, all critical constraints—including minimum funding per activity and total budget caps—were met to a practical extent, affirming the reliability of the resulting allocation for practical implementation.

To enhance reproducibility and accessibility, we highlight that while the current implementation relies on MATLAB's fmincon solver (Sequential Quadratic Programming method), comparable optimization can be achieved using open-source tools. Specifically, Python's SciPy library (scipy.optimize.minimize with method='SLSQP') offers analogous capabilities for addressing constrained nonlinear optimization problems. The optimization model's logic and structure can be entirely replicated in this platform. For transparency and future advancement, the authors plan to share core pseudocode and a minimal working version of the model through a public repository or supplementary materials.

The model's efficacy was confirmed by the algorithm's quick convergence, which finished optimization in two iterations after the first one. The excellent precision of the solution identified is indicated by the objective function's final value and the minimal changes in the variables. Even though there were a few minor infractions of the limits, the outcomes were nevertheless acceptable and satisfied the standards.

The level balance model and the linear programming model with maximum citizen satisfaction functions were two alternative approaches from related work that were compared in order to assess the efficacy of the suggested multi-criteria optimization model [3]. An examination of each method's outcomes and a comparison with the model used in this paper can be found below. The level balancing model divides the funds among regions and activity areas based on predetermined thresholds. This guarantees that the more crucial areas receive proportionately greater funding while each category receives the bare minimum permitted.

The results of the study showed a more balanced budget distribution. For instance, ecology in the Talgarsky district was allocated 3,253 thousand tenge, while technology in the Karasaysky district received 1,801 thousand tenge. Nevertheless, the absence of objective function optimization restricts the efficient use of resources. In conclusion, the level balance model promotes equitable distribution but overlooks strategic priorities and citizen feedback.

The second model, based on linear programming, aims to maximize citizen satisfaction by incorporating their preferences through voting data, with calculations performed using the simplex method. This approach maximized citizen satisfaction, though funding for lower-priority areas like culture and ecology was omitted. For example, healthcare in the Karasaysky district received 1,741 thousand tenge, surpassing the allocation in the level balance model. However, excluding funding for certain areas, such as infrastructure, introduces risks of imbalance. Despite not funding all categories, this approach proves highly efficient under resource constraints. The optimization model proposed in this study integrates multiple factors, including urbanization levels, strategic priorities, basic needs, and citizen satisfaction, with computations carried out using the quadratic programming method. The objective function value reached 18,482 thousand tenge, reflecting a balanced consideration of the criteria. All categories received funding, with healthcare accounting for 22.05% and transport for 21.11% securing the largest shares, while education at 7.03% and infrastructure at 11.26% received smaller portions. The model ensured adherence to both minimum and maximum budget constraints. To evaluate these models, a comparative table (table 5) was developed, highlighting the key common criteria across all approaches.

**Table 5.** Comparative analysis of models

| Criterion | Level balance model | Linear programming model | Multi criteria optimization model |
|---|---|---|---|
| Taking into account the opinion of citizens | Partly | Yes | Yes |
| Financing of all categories | Yes | No | Yes |
| The balance between criteria | Average | Low | High |
| Transparency | High | Average | High |

The best outcomes are shown by the multi-criteria optimization model, which guarantees effective budget allocation and a balance between several criteria. An all-purpose instrument considers social and strategic factors. The test findings demonstrate that the suggested multi-criteria optimization model is the most dependable and well-balanced approach to budget allocation. While a qualitative comparison with baseline models (e.g., level balancing and linear programming) has been conducted, a quantitative assessment using performance metrics (e.g., RMSE, MAE) could not be performed due to the lack of detailed, disaggregated budget data at the district and sectoral levels for 2022-2023. Government-reported allocations are typically aggregated, hindering direct alignment with the model's framework.

For future research, we intend to submit a formal data request to regional authorities to access detailed budget breakdowns, facilitating a thorough empirical validation and enabling the calculation of accuracy metrics such as prediction error and budget deviation. To compare the efficiency of budget allocation optimization methods in more detail, were analyzed two methods - the genetic algorithm GA implemented in Python using the DEAP library and the SQP method in order to compare the effectiveness of budget allocation optimization techniques in greater detail. Both approaches were used to solve a multi-criteria optimization issue that considers factors including urbanization level, strategic priorities, fundamental necessities, and citizen satisfaction. The comparison's goals were to determine the potential benefits and drawbacks of each strategy and assess how GA, as a stochastic method, can compete with the deterministic SQP in the budget allocation problem.

In contrast to the SQP, which depends on gradient-based convergence, the GA adopted a stochastic population-based strategy. It utilized a population size of 200, evolving over 500 generations, with an 80% crossover rate and a 5% mutation rate. Variability was introduced through a one-point crossover operator and Gaussian mutation. Constraint violations were penalized within the fitness function, employing dynamic scaling to maintain feasibility without excessively limiting exploration. The GA exhibited slow but consistent convergence, with notable improvements in early generations and stabilization in later stages. Although it required substantially more evaluations than SQP, GA offered robustness against local optima and achieved a high-quality solution with a final objective value of 18,520,000.00 thousand tenge.

The capacity of genetic algorithms to identify global optima in intricate multi-criteria issues makes them popular in optimization and financial allocation challenges the stock market forecasting problem demonstrates the ability of GA to adjust to complicated nonlinear dependencies, by optimizing model parameters, GA increased the prediction accuracy of stock prices [16]. The model is intended to be expanded in the future by accounting for indicator dynamics and uncertainty. To account for the imprecision and fuzziness of input data, it is especially recommended to combine fuzzy logic techniques with the evolutionary algorithm. One example of a successful hybrid strategy is the use of GA with fuzzy rules to optimize the cost-time-quality ratio of projects [17].

The effective optimization of KOSPI 200 index fund portfolios [18] shows that when Genetic Algorithms (GAs) were used for index fund management, they outperformed traditional optimization techniques in terms of portfolio performance. The creation of the GA-MSSR method, which maximized performance metrics like the Sharpe and

 

Sterling ratios to optimize high-frequency trading strategies, further demonstrated GAs' ability to adapt to complex nonlinear financial data and greatly increased the accuracy of micro-level price forecasting in volatile market conditions [19].

The genetic algorithm reached a value of 18,520,000.00 thousand tenge after modifying the parameters (increasing the number of generations to 500, increasing the penalties for violating restrictions, and normalizing the objective function) in contrast to SQP, which displayed the objective function value of 18,519,864.85 thousand tenge. Despite minor variations that reflect the stochastic character of the approach, the budget distribution derived using GA was found to be similar to the SQP results (figure 6):

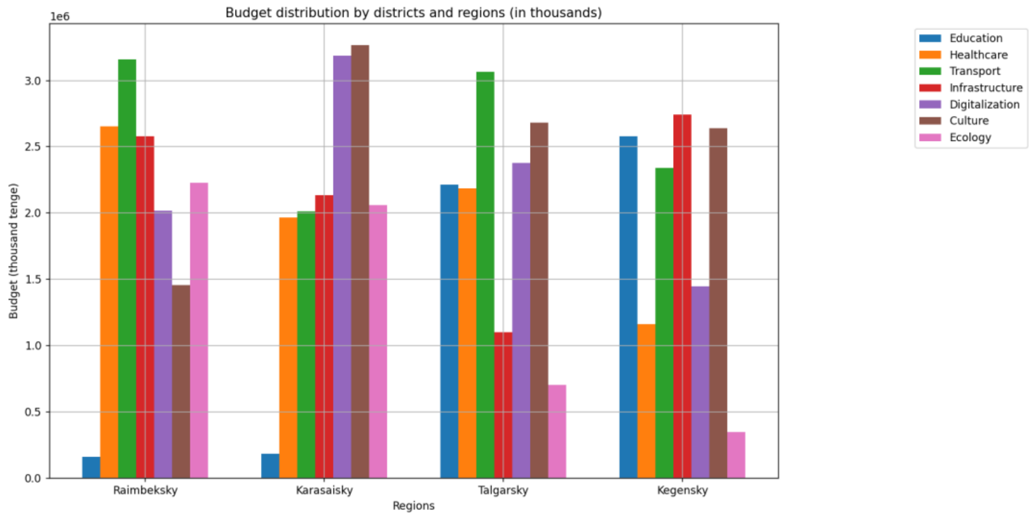


**Figure 6.** Distribution of the budget by districts and regions, obtained using GA

Similar to the SQP results, the graph indicates that the Kegen district, which has a high degree of urbanization, receives the biggest allocations of funds for infrastructure, while the Talgar district receives the fewest allocations for culture. However, because of the stochastic search for solutions, GA has a uniform distribution in other categories, including education. A comparative analysis of the methods is presented below (table 6).

**Table 6.** Comparative Characteristics of SQP and GA methods

| Characteristic | Methods | |
|---|---|---|
| | **SQP (MATLAB)** | **GA (Python, DEAP)** |
| Method Type | Deterministic | Stochastic |
| Objective Function | 18,519,864.85 thousand tenge | 18,520,000.00 thousand tenge |
| Convergence | Fast (less than 100 iterations) | Slow (500 generations) |
| Accuracy | High (deterministic approach) | Medium (depends on parameters) |
| Constraint Handling | Linear constraints built into fmincon | Via penalties in the fitness function |
| Advantages | Fast convergence, high accuracy | Global search, robust to local minimal |
| Disadvantages | May get trapped in local minimal | Random results, high computational cost |
| Applicability | Suitable for well-defined constraints | Suitable for complex tasks with uncertainty |

The SQP and GA methods can both solve the multi-criteria budget allocation optimization issue, according to a comparison study of the two methodologies. According to a comparative research, multi-criteria budget allocation problems can be successfully handled by both SQP and GA. As a deterministic method, SQP exhibits better accuracy and quicker convergence under well-defined constraints [20]. On the other hand, GA is excellent at examining trade-offs between goals and demonstrates resilience in situations involving a lot of uncertainty. It provides adaptable solutions for complicated issue structures [21].

SQP is better for problems with well-defined constraints because of its deterministic method, which shows higher accuracy and convergence rate. In turn, GA provides many approaches that can be helpful for examining criteria trade-offs, particularly when there is uncertainty. The suggested model's dependability is confirmed by the small discrepancy between the budget allocation and the objective function values (135.15 thousand tenge). It is advised that future studies take into account combining GA with fuzzy logic to account for data uncertainties and use more recent libraries like Pymoo to enhance constraint handling [22].

The optimization model allocated a total budget of 42,656,543 thousand tenge across four districts and seven Areas of Activity (AAs) in the Almaty region, utilizing a multi-criteria optimization approach. Healthcare (22.05%) and transport (21.11%) secured the largest shares, reflecting their high strategic priority and strong citizen support. Education received the smallest allocation (7.03%), indicating lower prioritization in voter and strategic metrics. All seven AAs were funded, showcasing the model's ability to ensure comprehensive sectoral coverage.

The model employed quadratic programming, with the Sequential Quadratic Programming (SQP) algorithm in MATLAB achieving an objective function value of 18,519,864.85 thousand tenge, converging in under 100 iterations. The GA, implemented via Python's DEAP library, yielded a slightly higher value of 18,520,000.00 thousand tenge after 500 generations, demonstrating robust global exploration. The negligible difference of 135.15 thousand tenge between methods confirms their reliability. SQP excelled in speed and precision under well-defined constraints, while GA proved effective for complex, uncertain problem structures.

Figure 1 illustrates balanced budget distribution across districts and sectors, aligning with regional priorities. Figure 2 confirms SQP's rapid convergence, with minimal step size and a gradient norm of 0.7016. Figure 3 highlights GA's similar allocation patterns, notably higher infrastructure funding in Kegen and lower culture funding in Talgar. Table 3 indicates strong influence of urbanization and population on allocations, while table 4 details precise budget amounts, consistent with optimization objectives.

## 5. Conclusion

A dynamic budget allocation model was developed and evaluated in this study using quadratic programming and multicriteria optimization approaches. The efficacy of the SQP method and a GA was compared. With a target function value of 18,519,864.85 thousand tenge with SQP, the model successfully allocates budgetary resources among districts and activity areas, according to the analysis and numerical experiments. GA produced a marginally higher value of 18,520,000.00 thousand tenge, a difference of 135.15 thousand tenge, demonstrating the robustness of both approaches.

In order to provide an accurate and well-informed resource distribution that meets real demands, the model takes into account important demographic data, urbanization levels, regional profitability, and public opinions expressed through voting. The model avoids financial imbalances and meets fundamental necessities while upholding strategic priorities by imposing minimum and maximum budget limitations. With the exception of a few minor allocation discrepancies, such as GA allocating 300,000.00 thousand tenge to education in Raimbeksky district compared to SQP's 296,622.18 thousand tenge, both approaches met most criteria. SQP, achieved rapid convergence within 100 iterations, while GA required 500 generations, reflecting its slower but more exploratory nature.

Although the proposed model exhibits significant potential for practical use, especially in participatory and multi-criteria regional budget allocation, it remains at the conceptual phase. A formal request for pilot testing has been submitted to the Ministry of Digital Development of the Republic of Kazakhstan for possible integration into the national eGov information system, with the authors awaiting a response. Consequently, while the results are encouraging, conclusions about real-world applicability should be considered preliminary pending validation in a policy implementation context.

The proposed model also highlights significant ethical concerns. The integration of publicly available demographic and socio-economic data with subjective weights for strategic priorities poses risks concerning transparency and perceived equity. Additionally, decisions guided by artificial intelligence and optimization algorithms may encounter public skepticism or political opposition, particularly if stakeholders lack a clear understanding of the underlying processes or perceive them as obscure.

To address these risks, future efforts will include the adoption of explainable AI techniques, enhanced transparency in data sources, and participatory approaches to validate weighting schemes. These measures are essential for fostering trust and ensuring ethical accountability in any potential deployment of the model within e-government systems.

These results can be used to guide budget management choices at the regional authority level, improving the effectiveness and transparency of budget planning. The comparison highlights the accuracy of SQP for clearly defined situations and the resilience of GA for complicated, unpredictable scenarios. To increase computational accuracy, the model can be extended to incorporate more complex economic and social factors, more places, more categories, and information such as population dynamics and long-term strategic objectives. The comparative analysis of SQP and GA shows that both are effective tools for optimizing government spending in resource-constrained environments, with each offering distinct strengths based on task requirements. As a result, the suggested approach guarantees equitable budget distribution, minimizes risks, and takes into account the interests of all stakeholders.

**6. Declarations**

## 6.1. Author Contributions

Conceptualization: S.G., S.S., Y.S. and A.A.; Methodology: A.A. and S.G.; Software: S.G. and A.A.; Validation: S.G. and A.A.; Formal Analysis: S.G., S.S., Y.S. and A.A.; Investigation: S.G.; Resources: A.A.; Data Curation: A.A.; Writing Original Draft Preparation: S.G. and A.A.; Writing Review and Editing: A.A. and S.G.; Visualization: S.G., S.S., Y.S. and A.A. All authors have read and agreed to the published version of the manuscript.

## 6.2. Data Availability Statement

The data presented in this study are available on request from the corresponding author.

## 6.3. Funding

The authors received no financial support for the research, authorship, and/or publication of this article.

## 6.4. Institutional Review Board Statement

Not applicable.

## 6.5. Informed Consent Statement

Not applicable.

## 6.6. Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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